

# Internally iterated ultrapowers

Karel Hrbacek

`khrbacek@nyc.rr.com`

The City College of CUNY, New York

It is obvious that the operation of taking an ultrapower of the set-theoretic universe modulo an ultrafilter  $U$  can be repeated finitely many times. H. Gaifman and K. Kunen elaborated a procedure for iterating ultrapowers along any linear ordering using these finite repetitions as building blocks (iteration with finite support). For countably complete ultrafilters and suitable well-orderings the procedure satisfies the Factoring Lemma: at each stage, one can think of the remaining iterations as being performed internally inside the universe obtained by iterating up to that stage. I will discuss how ultrapowers can be repeated hyperfinite number of times, study the resulting structures, and characterize them axiomatically. These structures will then be used as building blocks of an iteration procedure (iteration with hyperfinite support) where the Factoring Lemma holds for any ultrafilter and any linear ordering. The technique will be applied to show that ZFC can be conservatively extended to some strong nonstandard set theories.