Dense subfields and integer parts

Franz-Viktor Kuhlmann

fvk@snoopy.usask.ca

University of Saskatchewan, Canada

Answering a question asked at the Conference "Logic, Algebra and Arithmetic" in Teheran 2003, We show that every real closed (or more generally, henselian) non-archimedean ordered field L admits a proper dense subfield. This is interesting because any integer part of K will also be an integer part of L. If L is "small", then $L|K$ must be algebraic. However, there are examples of trdeg $L|K$ being any pre-assigned cardinal. In particular, if the natural valuation on L has no coarsest non-trivial coarsening, then trdeg $L|K$ can be any countable cardinal. At the same time our construction provides a counterexample to the following conjecture: "If a valuation v has no coarsest non-trivial coarsening and if the residue field with respect to every coarsening is henselian, then v is henselian."

We will also discuss the existence of integer parts and of truncation closed embeddings in power series fields. We show that Boughattas' ordered field which admits no integer part is not henselian. Finally, we show that there is a real closed field that has larger cardinality than the quotient fields of all of its integer parts. It is an open question whether there is a real closed field that is larger than the quotient fields of all of its integer parts, but has the same cardinality.