

# On the decidability of the real exponential field: candidates for a complete axiomatization

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## Abstract

In the paper [1], Macintyre and Wilkie proved the decidability of the structure  $\mathbb{R}_{\text{exp}} = \langle \mathbb{R}; +, -, \cdot, <, \exp, 0, 1 \rangle$ , under the assumption that the Real Schanuel Conjecture (RSC) is true. RSC says that given  $a_1, \dots, a_n \in \mathbb{R}$ , the transcendence degree (over  $\mathbb{Q}$ ) of  $\langle a_1, \dots, a_n, e^{a_1}, \dots, e^{a_n} \rangle$  is greater or equal to the linear dimension (over  $\mathbb{Q}$ ) of  $\langle a_1, \dots, a_n \rangle$ . In the paper [2], Berarducci and Servi proved that  $\mathbb{R}_{\text{exp}}$  is effectively o-minimal: there are recursive bounds on the number of connected components of  $\mathbb{R}_{\text{exp}}$ -definable sets. This result allows the authors to simplify the axiomatization proposed in [1] and to present new candidates for a complete axiomatization of  $\mathbb{R}_{\text{exp}}$ . Moreover, the results can be generalized to obtain similar decidability arguments (conditional on suitable conjectures) for some related structures (i.e. pfaffian expansions of the real field).

## References

- [1] A. Macintyre, A. Wilkie, *On the decidability of the real exponential field*. Kreiseliana, 441–467, A K Peters, Wellesley, MA, 1996.
- [2] A. Berarducci, T. Servi, *An effective version of Wilkie's theorem of the complement and some effective o-minimality results*. Ann. Pure Appl. Logic 125 (2004), no. 1-3, 43–74.

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