

# Non-standard aspects in model theory of modules

Carlo Toffalori

`carlo.toffalori@unicam.it`

Department of Mathematics and Computer Science

University of Camerino, Italy

Pure injective modules are a basic notion and play a relevant tool in the model theory (and in the algebra) of modules over a given ring  $R$ . Indeed every  $R$ -module has a "minimal" pure injective elementary extension (its pure injective hull). Moreover pure injective modules enjoy a nice structure theorem, as each of them decomposes uniquely as the hull of a direct sum of indecomposable pure injective objects, plus (occasionally) a further pure injective superdecomposable direct summand (i. e. a pure injective module without nonzero indecomposable summands). Not so much is known about these superdecomposable objects, although there are conditions ensuring, or excluding, their existence over certain classes of rings  $R$  according to some algebraic features of  $R$ .

Pure injective modules can be also viewed as nonstandard structures (indeed, pure injectivity is a weak form of saturation). In this perspective superdecomposable objects can be regarded as twice nonstandard.

We plan to treat these "nonstandard" features in the model theory of modules. In particular we will discuss the existence problem of superdecomposable objects over some rings related to Dedekind domains (such as pullbacks of Dedekind domains) and over integral group rings.

These results are a joint work with Gena Puninski and Vera Puninskaya.