ETHNOMATHEMATICS AND MATHEMATICS EDUCATION

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Copenhagen

Discussion Group 15
Ethnomathematics

edited by

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Introduction

by Franco Favilli

In the International Congress of Mathematics Education 10, which took place in Copenhagen (DK), 4 – 11 July 2004, Ethnomathematics was the theme of the DG 15.

The DG 15 Organising Team was appointed by the ICME 10 International Programme Committee and composed by:

Team Chairs: Franco Favilli (University of Pisa, Italy) and Abdulcarimo Ismael (Pedagogical University, Maputo, Mozambique).

Team Members: Rex Matang (University of Goroka, Papua New Guinea), Maria Luisa Oliveras Contreras (University of Granada, Spain) and Daniel Clark Orey (California State University, Sacramento, USA).

According to the discussion document, made available to the public with the contributed and accepted papers at *http://www.icme-organisers.dk/dg15/*, the DG 15 mostly aimed to provide a forum of participants to exchange their ideas and experiences in ethnomathematical research, particularly, those related to research on cultural aspects of mathematics education. Thus, the activities of DG 15 at ICME 10 aimed to discuss the following issues in the area of ethnomathematics:

1. What is the relationship between ethnomathematics, mathematics and anthropology, and the politics of mathematics education?
2. What evidence is there, and how do we get more, that school programmes incorporating ethnomathematical ideas succeed in achieving their (ethnomathematical) aims?
3. What are the implications of existing ethnomathematical studies for mathematics and mathematics education?
4. What is the relationship of different languages (or other cultural features) to the production of different sorts of mathematics?

The Organising Team was able to mobilise 15 contributed papers from all the continents. More than 50 people from more than 30 countries participated in the DG activities.

As regards the issues set for the discussion, even if no papers were completely or mostly related to the issue 4, in some of them the relationship among the different languages used in the classrooms and the different mathematical traditions and knowledge was clearly pointed out.

The DG activities developed in three sessions. In the first two sessions, the participants who had submitted a paper and were present at the Congress were given opportunity to give a very short oral presentation of their paper, in order to initiate and enhance the discussion. The final session was completely devoted to the general discussion.

As the reader can realize, the richness of the contributions and the variety of issues raised in the papers provide evidence of the great vivacity of the international community of a field of study – ethnomathematics – that keeps attracting an increasing number of scholars and which research outcomes are greatly contributing to a mathematical education more effective and, at the same time, respectful of the different cultures and values that are represented in most classrooms, throughout the world.

Acknowledgement: I am very grateful to Giuseppe Fiorentino for the editing of this book of proceedings.

Preface

by Ubiratan D'Ambrosio

Ethnomathematics, as a research program in the history and philosophy of mathematics, with pedagogical implications, received an important organizational framework in 1985, during the Annual Meeting of the National Council of Teachers of Mathematics of the USA, when the International Study Group on Ethnomathematics/ISGEm was founded, only one year after ICME 5, in Adelaide, Australia. ISGEm was initially chaired by Gloria Gilmer, from the USA, and a bi-annual Newsletter was established, initially under the editorship of Patrick J. Scott, then in the faculty of the University of New Mexico, in Albuquerque. The Newsletter has been published, regularly, since then.

Since ICME 6, in 1988, in Budapest, the ISGEm has promoted discussion groups in all the ICMEs. In 1998, the First International Conference on Ethnomathematics took place in Granada, Spain, organized by Maria Luisa Oliveras, and was followed by the 2nd ICEm, in 2002, in Ouro Preto, Brazil, organized by Eduardo Sebastiani Ferreira. The 3rd ICEm will take place in Auckland, New Zealand, organized by Bill Barton. We are now beginning to plan for a discussion group in ICME 11, in 2008, in Mexico, and we are already thinking about the 4th ICEm, in 2010. This means that every two years we have the opportunity to assemble the International Study Group on Ethnomathematics. And there is also the opportunity to meet each other and to organize sessions in national meetings, in different countries. This is a clear indication that our field is prospering in the international scenario.

It is for me a pleasure to write these words for the Proceedings of DG15, in ICME 10, which took place in Denmark, July 2004.

Being one of the elders of this new field of research in Mathematics Education, I feel it is appropriate to comment about my trajectory to Ethnomathematics. Probably, the first ideas came to my mind as a result of my work in the USA in the 60s and of my active engagement in the various social movements of the moment. In the decade of the 60s there was, world-wide, a clear non-conformist position with the *status quo*. Many social groups were claiming for a different future, and for this there was a need of new behaviour, new knowledge and new legislation. Things had to change.

The Affirmative Action movement, in the USA, focusing on racial discrimination, particularly of blacks, aimed at creating, for Afro-Americans, real opportunities for professional and social access. But, it is obvious that only opening the doors and letting in those people who were, hitherto, rejected, is not enough. Once inside, they have to feel comfortable. They can not have the feeling that they do not fit in that ambiance, that they are culturally strange and unprepared for the new opportunities being offered. The pipeline of access may continue to be clogged, although in a later stage.

This has to do with the lack of recognition, indeed the denial, of previous, culturally rooted, modes of behaviour and systems of knowledge. This was particularly noticed in the universities. In the 60s I was teaching in a major university in the USA [SUNY at Buffalo], and I was in charge of the Graduate Program in Pure and Applied Mathematics. One day, I received a laconic note from the Dean’s Office saying that the new admissions should include 25% black students. This means 15 black candidates for doctorate in a total of 60 new students admitted to the program. This was part of SUNY’s policy of establishing a quota for admission of black students, in all its undergraduate and graduate programs.

It soon became very clear to me that just opening the doors and letting some students in do not make the guest comfortable when inside the house. Admission to the program was necessary, but it was not enough. For centuries, their traditional systems of knowledge had been ignored, rejected, denied and even suppressed. It was then necessary to restore cultural dignity and pride, anchored in traditional systems of knowledge. Since curriculum is the strategy for educational action, something had to be done with the curriculum. I understood that there was necessity of adopting a new dynamics of curricular change, based on new objectives, new contents and new methods. But these three components, objectives, contents and methods, had to be considered in absolute solidarity. It is impossible to change only one without changing, accordingly, the other two. This holistic concept of curriculum could not be based on a rigid, pre-established, set of contents. Everything had to be changed, in solidarity, reflecting not the epistemology and internal organization of the disciplines, but the cultural history of the students.

In the late 60s and early 70s, I had the opportunity of participating in several projects in Africa and in Latin America and the Caribbean, sponsored by UNESCO and the Organization of American States. In the then called “undeveloped countries”, I recognized the same pattern of misunderstandings, which I had witnessed in the Affirmative Action movements in the USA New ideas about education were, thus, building up in my mind. A holistic concept of curriculum seemed, to me, to be the crucial issue.

The opportunity to arrange all these ideas came when I was invited to prepare the discussion paper for a working group in ICME 3, held in Karlsruhe, Germany, in 1976. I was asked to prepare a paper on “Why teach mathematics?” with and alternative title: “Objectives and goals of mathematics education”. There is an almost unanimous understanding, among mathematics educators, that Mathematics is the body of knowledge originated in the Mediterranean Antiquity, organized by the Greeks, which expanded to European countries in the later Middle Ages, and which began to take its current academic form in the 14^{th} and 16^{th} century. This is the body of knowledge which prevailed in Europe and which, in the age of the great maritime discoveries, the conquest and the establishment of colonies, was imposed to the entire planet. In the encounter of European with other cultures, existing indigenous knowledge was selectively expropriated by the conquerors and colonizers, but at the same time they were ignored, rejected, denied and even suppressed as coherent systems of knowledge of the conquered nations. The systems of indigenous knowledge included, particularly, ways of dealing with space and time and different ways of observing, classifying, ordering, comparing, measuring, quantifying, inferring, inventing, plus coherent systems of explanations of facts and phenomena, based on sophisticated founding myths. These are the basic supporting elements of every cultural system and include mathematical ideas present in all these systems. Particularly in the Western cultural system, where they are organized as what is called Mathematics.

Based in these considerations, I suggested, in my paper for ICME 3, a broader historiography [including the mathematics of non-mathematicians], as the basis of a reflection about Mathematics and society, particularly about the question of PEACE, and about the relations between Mathematics and culture.

The pedagogical proposal associated with this asked for a holistic concept of curriculum, and for the recognition that Mathematics acquired by societies which are not in the main stream [consumers, not producers, of Mathematics] caused a cultural strangeness of Mathematics, with damaging consequences for education. These ideas were further developed and presented in a talk I gave in ICME 4, which took place in Berkeley, USA, in 1980.

The ground was thus laid for my plenary talk in ICME 5, in 1984, in Adelaide, Australia, entitled “Socio-cultural bases of mathematics education”. In this talk I presented some theoretical considerations, motivated by examples of indigenous tribes and labourers communities in the Amazon basin, dealing with their rituals, as well as with their daily activities, in which mathematical ideas were evidently present. The theoretical reflections and the examples presented culminated into the concept of ETHNO+MATHEMA+TICS.

The main issues raised in this talk can be synthesized as a broad historiography [which includes a form of mathematics done by non-mathematicians, that is, by common people], plus the Cultural Dynamics of the Encounters, throughout history. This implied the need of recognizing the political dimension of Mathematics Education, going back to the expansion of the Empires since the 15th century. Of course, this asked for an analysis of the State of the World today.

I believe that the main concerns of people all around the World are, nowadays:

- national security, personal security,
- government/politics,
- economics: social and environmental impact,
- relations among nations,
- relations among social classes,
- people's welfare,
- individual sanity of body and mind,
- the preservation of natural and cultural resources.

These concerns affect, profoundly, Mathematics Education. In fact, Mathematics, mathematicians and mathematics educators are deeply involved with all these issues.

Indeed, these issues are intrinsic to the phenomenon life, affecting humans as individuals and in their relations other individual, and how individuals, organized as societies, deal with their environment.

Most intriguing questions are: why people destroy their environment? And why do people interrupt their own life [for example, practicing suicides], and the life of others, through homicides, bombings, poisoning, and even legal executions [for example, accepting the institution of death penalty]? Why arrogance, iniquity, bigotry is so common? Regrettably, history shows that mathematics is intrinsically involved with all these acts against life in its broad sense.

I believe no one will deny that Mathematics, as an organized set of concepts and ideas, is the most universal mode of thought. And no one will deny, either, that survival with dignity is the most universal human desire.

I feel it is an absolute priority to look into the relations between these two universals. I consider my mission, as a member of an organized society, to reflect on the responsibility of mathematicians and mathematics educators in offering the elements to respond to this priority.

We all know that Mathematics is powerful enough to help us to build a civilization with dignity for all, in which iniquity, arrogance and bigotry have no place, and in which threatening life, in any form, is rejected. For this we need to restore Ethics to our Mathematics. History tells us that up to the end of 18th century, Mathematics was impregnated with an ethics convenient for the power structure of the era. So, it makes sense to talk about ethics in mathematics.

I believe Ethnomathematics can help us to reach the goal of Mathematics impregnated with Ethics. And what is, for me, Ethnomathematics?

Ethnomathematics is a research program in the history and philosophy of mathematics, with pedagogical implications, focusing the arts and techniques [tics] of explaining, understanding and coping with [mathema] different socio-cultural environments [ethno].

The pedagogical strand of ethnomathematics must answer the major goals of education:

- to promote creativity, helping people to fulfill their potentials and raise to the highest of their capability;
- to promote citizenship, transmitting values and understanding rights and responsibilities in society.

A convenient pedagogy for ethnomathematics includes projects and modelling.

The current scenario calls for a critical views of mathematics education as the result of stressing its political dimension, but this meets the resistance of a nostalgic and obsolete perception of what is mathematics. This is supported by a perverse emphasis on evaluation. This creates an atmosphere unfavourable to Ethnomathematics.

The resistance against Ethnomathematics may be the result of a damaging confusion of ethnomathematics with ethnic-mathematics. This is caused by a strong emphasis on ethnographic studies, sometimes not supported by theoretical foundations, which may lead to a folkloristic perception of ethnomathematics. An illustration of this misunderstanding is the misleading title of a special issue of the French journal *Pour la science,* which was entirely dedicated to Ethnomathematics. The title of the special issue is *Mathématiques éxotiques*.

My proposal is to give more attention to the theoretical strand of the Program Ethnomathematics, as a research program in the history and philosophy of mathematics [both Mediterranean, Western and non-Western], with its pedagogical implications.

I am so happy that much of what was presented in this DG15, during ICME 10, addresses this proposal.

Foreword
by Bill Barton

Moving Forward

The trajectory that is ethnomathematics has moved in many ways since those early inspirations of the 1980s that Ubiratan recounts in his foreword. An increasing number of studies – as illustrated in these Discussion Group proceedings – and an increasing number of people becoming interested in ethnomathematics means that there is a wide diversity of opinions and motivations for work in the field. We celebrate diversity, yet we continue to be inspired by Ubiratan D'Ambrosio's vision – and the metaphor that he gave us in the plenary session of ICME-10 is one of his most powerful.

His counter to those who seek to marginalise ethnomathematics by claiming that it is anti-mathematical is to affirm that mathematics is truly beautiful and fundamental. He describes it as the "dorsal spine" on which our interaction with the world is built. Mathematics always was, and will continue to be, the backbone of human knowledge, the way in which we make sense of quantity, relationships and space. The question that he poses to us, however, is the way in which the body is built on this spine: will it be an ugly travesty of what knowledge can do for humankind, or will it be as beautiful as the stark structure of the spine and move us forward into a more peaceful and respectful future?

Ethnomathematical work – as exemplified in DG-15 – all contributes, in essence, to bringing the human dimension back into mathematics. Those studies that describe cultural activities from a mathematical point of view are all demonstrating the idea that much human activity has, from the earliest times, been mathematical in form – that mathematical activity is not the preserve of mathematicians. Those studies that are investigating the use of ethnomathematical ideas in the classroom are exploring how mathematics that is associated with cultural values and practices can have a role in the education of future generations. Those studies that are questioning the nature of mathematical concepts in different cultural contexts are setting up a dialogue between conventional mathematics and more humanly connected forms of activity.

The dorsal spine is made up of many bones finely crafted for distinct purposes and fitting together into a structure of brilliant ideas. How are we to go about ensuring that the beautiful dorsal spine produces the kind of body of human relations we seek? One component of this task is to fully understand how mathematics (and science) has come to be so disassociated from human relations that it is possible for people to cheer and celebrate the accuracy of a missile without any acknowledgement of the human tragedy that this accurate detonation represents (to take another of Ubiratan's images).

Mathematics was not always so disassociated. Newton, for example, regarded his work as discovering God's work (Brewster, in press), and Arab mathematicians (for example, Omar Khayyam in *Treatise on Demonstration of Problems of Algebra*) wrote long prefaces about mathematics as part of the glory of Allah. As well as the spiritual, the philosophical is closely linked with the history of mathematics: Plato, Descartes, Russell and many others. How have the values of mathematics (rationalism, objectism, control – see Bishop, 1990) come to divorce themselves from the purposes for studying mathematics? Answering this question might help us decide how to reconnect.

Such an investigation echoes again D'Ambrosio's definition of ethnomathematics as a project in the History and Philosophy of Mathematics. We must find a new way to look at, and to understand, the mathematics that has come to dominate schools, universities and research in the 21st century. We need a new historiography.

Let me put forward a small idea in this project. It seems to me that part of the explanation for the dissociation of science and mathematics from human values lies in the very connections between mathematics and philosophy, and between mathematics and religion.

Philosophy, as a search for truth, has long looked to mathematics for certainty. Morris Kline's book "*Mathematics: The Loss of Certainty*" (Kline, 1980) documents mathematics' ultimate failure to carry this burden, and yet its legacy continues. Mathematics is still regarded as some kind of ultimate knowledge about which we can be certain, and which cannot be challenged. One plus one is indubitably two. Ultimate knowledge cannot possibly, of course, have human origins, therefore mathematics must be superhuman, it must be beyond human creation. Although the argument is not explicit, perhaps the legacy of philosophical expectation for mathematics is to elevate mathematical knowledge out of the sphere of human responsibility: it becomes ok – no, more than that, it becomes preferable – to do mathematics as free as possible from human preconceptions, social motivations, language dependence, or cultural preference.

In the face of the evidence that mathematics is a human creation dependent on all these things (for example, my own studies amongst research mathematicians), mathematics has continued as if it is ultimate knowledge.

Religion's legacy has a similar effect. Early mathematicians thought that their work to discover the greatness of the god they worshipped, a proof, if you like, that there was a greater being, or a wider reality than that offered by the human mind that could be deceived. The immense power (and beauty) of the mathematical edifice generations of mathematical thought has produced only serves to enhance this perception. Mathematics is clearly greater than what can be encompassed by any single human mind. Thus, when the secular nature of academic endeavour has become predominant in much of scholarly life, the awe inspired by mathematics is such that mathematics itself becomes the god. The consequence of its worship over and above human concerns is the very alienation that Ubiratan has described to us.

If these accounts of the separation of mathematics from human values have any legitimacy, then perhaps, in them, we can find ways to construct the new historiography? Acknowledging the philosophical and religious legacies might be a first step in reclaiming mathematics as a human endeavour that is in dire need of reconnection to human values and human ethics.

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OCCURRENCE OF TYPICAL CULTURAL BEHAVIOURS IN AN ARITHMETIC LESSON: HOW TO COPE?

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In honour to Guy Brousseau

I. Problems.

If mathematics is a universal science, then the specific conditions of its diffusion and the patterns which allow for it must be specific too. But what is true for mathematics and its didactic could not be asserted as far as the phenomena of teaching *lato sensu* are concerned. In fact the didactic conditions do not fill all the conditions of teaching, far from it. The anthropo-didactic approach is interested in the phenomena related to that non filling field; more precisely it is interested in the crossing between the didactic and non didactic conditions – particularly the anthropological conditions *lato* and *stricto sensu*.

This approach is not situated exactly in the classical view of ethnomathematics as constituted with the other ethnosciences within Anthropology. If we refer to two relatively recent works, by M. Ascher (1991) and U. D'Ambrosio (2001) – the former more prospective, the latter more ideological and polemic – ethnomatematics most often consists in identifying an explicit mathematical activity (consequently some real cognitive work) in social practices that are clearly itemised, even ritualised. These practices concern numeration, art, craftsmanship, music, games, organisations of parenthood… and they sometimes may be transposed into the formal school curriculum. In a special issue of *Educational Studies in Mathematics*, Bishop (1998) sums up rather well the theses establishing a link between Ethnomathemtics / Mathematics universality / Mathematics education.

> The thesis is therefore developing that mathematics must now be understood as a kind of cultural knowledge, which all cultures generate but which need not necessarily 'look' the same front one cultural group to another. Just all human cultures generate language, religious beliefs, rituals, food-producing techniques, etc., so it seems do all human

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> cultures generate mathematics. Mathematics is a pan-human phenomenon. Moreover, just as each cultural group generates its own language, religion belief, etc., so it seems that each cultural group is capable of generating its own mathematics. Clearly this kind of thinking will necessitate some fundamental re-examinations of many of our traditional beliefs about theory and practice of mathematics education. (Bishop, 1998)

Closely related to the effective social practices of formal teaching anthropo-didactics is not interested in metaphysical and all-covering questions so much. Contemporaneous poststructuralist Anthropology is more modest and pragmatic: cultures are not those isolated machines generating algorithms to which people would conform unknowingly logical. By rubbing against each other and taking into account the interests of subgroups with changing configurations within themselves, cultures do their best to preserve multiple identities – exit the theory of the great gap between global and segmented societies – through more or less successful interbreeding[1].

To speak plainly anthropo-didactics considers school situation as doubly structured, both 1) didactically and 2) anthropologically:

1) Any school situation aims at making pupils take over a certain number of social practices – gestures, specific ways of dealing with problems and accounting for them. The use of these practices by the pupils must conform to those of a given disciplinary community: one does not learn or do Maths the same way as poetry or mechanics. Yet: i. these practices are not directly teachable – if they were there would not be any ‘didactic problem’; ii. the transposition into the school field of the social frames of those practices and the resort if usual to “make believe” do not directly favour the takeover of knowledge by the pupils. As already shown (Clanché & Sarrazy, 1999) such a form of teaching seems to arouse more difficulties than it actually solves. The situations of teaching are not isomorphous to the social situations of production and use of knowledge.

   That is why we think that the ‘culturalist’ introduction – ie. without any *a priori* didactic reflection– of social ethnomathematical practices

[1] “[Culture or context is] a set of attitudes and thoughts with their one logic but that a **given situation** can bring together to the heart of a same phenomenon.” (Bensa 1996).

exogenous to the school system may miss its aim and 'folklorise' the local culture one wants to legitimate.

2) If the Theory of Didactic Situations (Brousseau 1997) makes it possible to identify and study the specific conditions of the diffusion of mathematics knowledge, as well as to foresee the effects generated by such or such situation, its focalisation tends to leave some anthropological conditions aside. These conditions are equally specific of the school frame in so far as it is through these conditions that pupils can get to belong do it and therefore build up their affiliation by adopting the same approach to deal with mathematical questions. The anthropological conditions are not set up once and for all like the unique setting for a theatre performance. They occur haphazardly like the particular configuration arising from the pushing of an unexpected card onto the baize during a game – such pushing being authorized by the constitutive rules of the game (Searle 1969).

The observation of a lesson during which one of us (B.S.) played the part of the teacher will illustrate our talk.

II. Conditions of the observation

The lesson analysed is an introduction of Euclidian Division: the research of the 'q' quotient by building up the multiples of the divisor, also called 'multiplication with blanks': D = q x d. The observation took place in an elementary school form ("CM2", 10 years-olds) in a Melanesian school (New-Caledonia, Northern Province, Paicî linguistic aera).

On top of the novelty brought along by the object of teaching, the lesson set up two breaks as regards the usual school custom of the pupils: i. a change in the teachers: the mistress agreed to have one of us 5 (BS) take charge of the class after a week's participative observation; ii. A change of didactic culture: it was probably the first time the pupils had to face an open situation (vs. a lecture). In that situation they were required to establish the validity of their decisions by themselves. We will call "devolution of the proof" the most important change by reference to one of the central concepts of Theory of Didactic Situations.

> Devolution is the act by which the teacher makes the student accept the responsibility for a learning situation or for a problem, and accepts the consequences of this transfer of this responsibility.

> Doing mathematics does not consist only of receiving, learning and sending correct, relevant (appropriate) mathematical messages […] Proof will be formulable only after having be used and tested as an implicit rule either in action or in discussion." (Brousseau 1997).

This was when the anthropo-didactic phenomena appeared.

III. Lesson

BS writes on the board:

> *Since Martine was born her parents have been giving her two exotic fish for her birthday. This year she counts 24 fish in her aquarium. How old is Martine this year?*

After the pupils have worked in small groups they give the group's solutions

SOLUTIONS (on the board)

Impossible sum **12 years** **48 years**

BS asked the children which solution was right, to **give the proof** and to **convince** everybody:

BS - Now you have to prove that your solution to Martine's problem is the best; Not only must you find it, but prove it! But you do not have to prove it to me because I know the solution, but can't say it. So you must convince the others.

In the following talk, mainly four pupils participate:

> Glenn and Rezzia hold the **internalist position** and defend the solution **12 years**
>
> Fabrice et Eddy hold the **externalist position** and defend the position **impossible sum**

BS's task **is not to kill devolution** by: i) keeping conversation on a horizontal axis, ii) keeping the mathematical meaning, iii) making knowledge progress.

Apparently the sequence is a sort of match, but BS is not a referee, he is rather a coach for both teams. His goal is not to see a team defeat the other, but to see the good solution (mathematical knowledge) recognized as true by all the class.

This is what we will see in the following sequence; BS could have taken the things in hand several times and convinced the pupils quickly and superficially. His attitude to devolution has always prevented him from interrupting the devolution. But at the same time, his attitude has not non-directive, he has the responsibility to make knowledge advance. He changed the situation, allowing “losers” to not “lose face”.

This pseudo match can be divided into 7 sequences :

1. Local institutionalisation of the idea of proof
Not enough time

2. Devolution of proof I: Externalisation
Fabrice and Eddy give the external reasons for their position.

FABRICE [turning to BS] – We can’t because **some fishes have already left**.

BS - You don’t have to prove it to me, you must convince your friends. Let’s do it another way. Who thinks Fabrice is wrong?

An internalist, Rezzia, protests and proposes to do an operation.

BS pretends not to have heard anything. He could have then concluded and blocked the devolution of the proof.

3. Devolution of proof II: First Internalisation followed by Switching
Not enough time, but:

FABRICE – The fishes can die!

REZZIA – The text doesn’t say so.

BS agrees – The text doesn’t say so. Let’s say the fishes don’t die

From the point of view of Biology Fabrice is absolutely right! An exotic fish cannot live more than 3 years in an aquarium. Let us note internalisation of Rezzia and over the BS’s reframing which avoids a long unless debate!

4. Pseudo proof: Vote
Facing oppositions BS asks the pupils to discuss how to find accordance. The pupils propose to vote. **This decision, a mathematical crime, becomes an excellent didactical decision!** The result of the votes is:

Solution 12 years	15
Solution impossible sum	3

Those supporting 12 celebrate it!!!

BS – Okay, I don't agree because it means there are still three children who don't agree. So, we go on until everybody agrees.

5. Devolution of proof III : Debate Externalist vs. Internalist

BS gives 5 minutes for each group to prove that they are right. The discussion continues around the table and gets harder.

There starts a typical cultural behaviour…

EDDY – But her mom, since Martine was born, her mom she can be dead…

GLENN [*From his place*] – Her mom, well, she's not old yet!

EDDY – When she was born! But it means that she is old now. When occurs at two, she started getting old, at three and at six, seven, at seven **she starts dying.** We cannot know if we calculate the number of fishes, we cannot calculate the same way. You must calculate these things the same way. The GIRL, she would be fed up with fishes, maybe she'd prefer toys now and then she decides mom to buy a car or bike… When she is six, the fishes they will grow up, they will be too big in the aquarium, so she is obliged to eat… Or, maybe cats have to eat as well… Maybe somebody can break the aquarium…

Eddy's attitude:

- Is he a social actor defending his place in a prestige fight against Glenn and Rezzia?
- Or does he get into a mathematical argument knowingly?

The anthropo-didactic answer is: **both.**

Eddy is a good pupil opposed to against Glenn – class leader and a good pupil too; Rezzia, a bright shy girl, a reverend's daughter ? The three of them come from the same tribe. An anthropological argument: adults and children must not push themselves forward in front of an adult whose custom rank is higher. Eddy would probably have had another attitude in front of the teacher. The situation of devolution allows him to push himself forward in front of his peer group.

Apparently Eddy tries not to lose face, but in an anthropo-didactic way.

- Anthropo : "*she starts dying*"

In the Kanak culture death is not an accident but a process: one gets old so he can be dying: Martin's mother may have died since…

- Didactic : "*You must calculate these things the same way*"

Eddy uses his didactic memory as an argument – the teacher has done a few meta didactics lessons.

6. Proof & Validation

BS is getting tired and asks if someone has found something. A contingent element rewards him.

" I have!". The internalist Rezzia has taken plastic cubes which were on the table and she has started to gather cubes by twos!

BS – Come, come! In that way everybody can see! Show your method.

JENNY – Two cubes is one year [*takes two cubes*], four cubes is two years […]

BS - When she is one, EDDY, how many fishes are there in the aquarium?

SEVERAL PUPILS – Two, two!

BS – Do you agree with this or don't you EDDY? Yes?

EDDY – Yes!

BS – So, we can make an aquarium.

For an aquarium BERNARD takes a chalk cardboard.

BS [*throwing cubes and stones one after the other*] – At three, two fishes, four, two fishes, five two fishes, I'm not cheating! Six, two fishes, seven, two fishes, […] twelve, two fishes. I have cheated or not?

SEVERAL PUPILS – No

[…]

BS –There are twenty-four fishes… And Martine, how old is she?

SEVERAL PUPILS – Twelve years

[…]

BS – Do you agree EDDY now or don't you?

EDDY – I do.

7. Conclusion & Institutionalisation

In the end BS institutionalises on columns the division by the method of quotients on the board.

Total duration: 45 minutes

IV. General conclusion

Such is the teachers' day-to-day practice, even if they are didacticians!: they have no other choice but to teach their pupils mathematics by "doing mathematics". As for the pupils they have no other choice but to try to take over as "way of seeing", in the Wittgensteinien meaning. This double necessity sets the question of teaching in the middle of the anthropological field: "show" to "see like" – eventually agreed on by Eddy – imposes itself to the pupil as an necessity. As Conne & Brun (1990) say "Conceptualization is due to follow initiation". Let us repeat the paradox of devolution:

> The teacher wants the student to want the answer entirely by herself but at the same time he wants – he has the social responsibility of wanting – the student to find the correct answer. (Brousseau 1997).

This statement "The teacher wants the student to want" might makes us smile! Yet Eddy's episode shows that the teacher cannot oblige students to "see like". In the *Philosophical remarks*, § 35, Wittgenstein says that the questions: what's a number? what's the meaning? what's the number one? give us "mental cramps"!

Learning mathematics amounts to learn a language game. Understanding the process of mathematics education amounts to describing and identifing the anthropological conditions in which those games were born and disappeared.

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MULTICULTURAL CLASSROOMS: CONTEXTS FOR MUCH MATHEMATICS TEACHING AND LEARNING[1]

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Many if not most mathematics classrooms are micro sites of multiculturalism. Hence the notions of ethno mathematics are in play whether it is acknowledged or not. However the fact that there are often multiple cultures and languages represented means that the learning and teaching carried out in these classrooms is more complicated. In this discursive paper, some pertinent themes first discussed during a Thematic Working Group of the third Conference of European Research in Mathematics Education are elaborated. In particular the notion of multiple contexts of classrooms, because of the variety of possible combinations of cultures and languages being present, in emphasised as potentially being an important factor that has not been recognised so far in research.

In many parts of the world, if not most, the great majority teachers may now expect to work with pupils from ethnic, linguistic, and cultural groups distinct from their own. Cultural, linguistic, political, and social issues in mathematics learning have until recently been often seen as distant to and have little impact on the teaching and learning of mathematics. But the problems of ‘others’ that are ‘different’ from ‘us’ are now a reality. If mathematics education is to become an equitable practice, then these issues need to be seriously addressed in most of our classrooms. That is, there is a continuing need for research that takes seriously an understanding of the complexity of the teaching and learning of mathematics in multicultural situations, and the possible benefits this may have for a more equitable society.

Cultures can be understood as knowledge, beliefs and conceptions, in this case, about particular mathematical situations. However they can also be understood as a pattern of meanings, historically constructed and socially transmitted, that are embodied in symbols and language, through which

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human beings communicate, perpetuate and develop their knowledge and their understanding of life. With the presence of different cultures understood in these ways within single classrooms, that is multiculturalism in a microcosm, there clearly are challenges for pedagogical traditions of mathematics teaching. Multiculturalism also has interesting ramifications for the broader school contexts within which individual classrooms sit, such as the forms of socialization that organization and management such schools promote, that in their turn clearly flow into the classroom.

Often the researching of the teaching and learning of mathematics in multicultural situations is closely linked to the phenomena of migration. Migration can no longer be considered only as an 'emergency situation'. There is a global increase in the number of refugees, resulting often in migrant adults and children living in places where the language and the culture are different from that of their origin. Hence the contributions that this research makes can be seen in the context of a growing social global phenomenon that many societies see as a problem rather than as an opportunity. Such a context is important to consider given the traditional narrow focus that many teachers of mathematics at all levels hold.

In this discussion three sites of impact and generation have been noted on the way through: the classroom, the wider school, and the broader society. The main site for this paper's discussion will be in the classroom, but the other interacting sites cannot be ignored. It is well to remember the dynamism of the situation that is the subject of discussion.

There is still an assumption among many mathematics educators that mathematics is free of culture, beliefs and values. This assumption holds that mathematics can be taught in the absence of a common language because it is 'universal'. For many people, the common understanding of the learning context of a student is 'monolingual', belonging to the dominant culture, and having the social habitus of middle class. Such an assumption does not countenance that there is a reciprocal dynamic; learning is influenced by language and culture, but as well, language and culture influence what is taught. For them, the mathematics classroom is not the best place to learn the language and the norms of the school. It is taken for granted that students have already a mastery of the language of the instruction and its subtleties, and this is some how automatically linked by the students to the discourses of different subject taught in the school. It is also a common assumption that the students know the 'norms' of the school. But such is just not the case for many students, particularly those from migrant communities. For example, it is particularly difficult for children from a non-western background, migrating to a western or westernised country to learn western mathematics when it is understood as part of western culture (Alro, Skovsmose, & Valero, 2003; Favilli, Oliveras, & Cesar, 2003; Moreira, 2003). Further, this mathematics curriculum, embedded in the wider school curriculum, is intended for monolingual, middle class students, belonging to the dominant social group. It is this context, not that traditionally assumed to be the case, within which out research must progress.

Indeed it is now clear that mathematics teaching and learning is a process where cognitive, affective, emotional, social, cultural and linguistic factors are deeply intertwined (Bishop, 1988; Ellerton & Clarkson, 1996; Lave, 1988). Further, the multiple links among these factors makes the teaching of mathematics a complex task, which becomes even more complex in multilingual or multicultural situations. In a classroom, neither the teacher nor the researcher may now assume that they are part of, or with, a homogenous group. Indeed there should be a recognition by teacher and researcher that there is a great heterogeneity amongst the several multilingual or multicultural situations that can, and probably is, present in any one classroom. This complexity of the research contexts in our domain requires the use of a multi layered theoretical perspective, and a great sensitivity towards the different cultures that may be present.

For a long period, most of the research concerning ethnic, cultural or linguistic minorities and their learning of mathematics focused on the mathematical achievement of those groups. It is only recently that researchers' interests have turned to the understanding of how and why this occurs for most such students, who normally obtained low achievement scores in mathematics. And why it is that there are very interesting exceptions for a particular small group of such students (Clarkson, submitted paper). This new direction for research has not been at the expense of a focus on achievement. The societal need for high achievement in mathematics is normally present when there is an emphasis on schooling, and hence this outcome can not be neglected by mathematics education research. The new direction is more of opening up, another parallel line of investigation, with the belief that both are interrelated. However, there is also an understanding with this new direction that 'achievement' should no longer been looked upon as the sole arbiter of whether students are 'succeeding' or a particular program is 'performing well'.

So gradually the notion of 'achievement' as the ultimate measure of quality in all things is coming under challenge, although whether this change can be brought about in the understanding of society in general is more problematic.

One interesting example comes from work with small groups in classrooms. In the search for an understanding of the mathematics learning of individuals belonging to groups that are culturally different to the dominant one, the idea of ‘participation’ seems to be crucial. Participation refers to both participation in the mathematical verbal conversation and in the broader mathematical discourse that takes place in the small group, within the classroom, as well as participation in the wider school culture (Clarkson, 1992). All seem to be crucial. Participation is an essential process for inclusion. It has to be mediated at least in part by the teacher, and has to take into account both the students’ background and foreground. The formal mathematics education of an individual requires his/her participation in an institutional network of practice where empowerment, recognition and dialogue are tools to face conflict in a positive way. Conflict should be understood not only as cognitive conflict, but also as cultural, social and linguistic conflict, and in this broader sense, it must be seen also as a tool for learning. Indeed it may turn out to be the critical strategy for learning. Once this type of thinking is entered into, achievement seems to be a very gross measurement for a conglomerate of interconnected processes that function when a student is learning.

Turning back to a description of multicultural mathematics classrooms, it is useful to think a little about one aspect of this situation, the variety of language possibilities that may be present. There are a variety of non-homogeneous linguistic situations that fall under the umbrella of multicultural situations. Such situations include classrooms where the language of instruction is different from the first language of the students, for example the teaching of recently arrived immigrant students. At least in some places, for example southern states of USA, there can be classes of students who speak the same language, although it is a non English language (Cuevas, Silver & Lane, 1995). However the situation can be more complicated than this. In some European countries the new influx of migrants mean that schools are admitting students who come from a number of different language groups, and they sit with students who speak the language of instruction as their first language. In some other countries such as Australia there is yet again a different variation. There is a continuing flow of new migrants from different language groups being added to older migrant families who speak other languages. For example many schools who still have first or second migrant families who came from southern European countries and still speak Greek, Italian, Croatian, etc. in the home are being joined by students from Vietnam, India and Cambodia, but the teaching language for all is English (Wotley, 2001). A further scenario is when the teaching of mathematics may be in a language, which is not the first language of the teacher or students. In Papua New Guinea this happens where from year 3 on the teaching language is English but students and teachers may well speak quite different languages in their homes. And yet another situation is found in Malay schools in Malaysia. There the new policy is for mathematics and science to be taught in English, but all other subjects are still taught in the language common to both teacher and students, Baha Malaysian. How communication and learning takes place when the languages spoken are not shared, how the fluency of the language of instruction is related to the mastery of the broader notion of mathematical discourse, how using a particular language is linked to different ways of learning, are all questions that need further exploration. But the quite different possible contexts in which such questions may arise has as yet not been taken seriously in our research (Clarkson, 2004).

The recognition of such situations when producing insightful research questions may well prove to be important. However there are other approaches that might also prove to be useful, such as the mapping out different types of broad contexts within which research questions focussed on multilingual mathematics classrooms could sit. Two such sets of contexts are noted here. The first is the complexity of language linked to mathematics education. This gives rise to at least four practical issues:

- different ‘levels’ of language (families of languages, distance between languages)
- different language contexts (indigenous, multilingual, immigrants)
- contexts within language (for example, speaking, listening, writing, reading) as well as the immediate context (conversational compared with academic)
- content realities (cultural, social, political).

There are also at least four theoretical issues that seem to be relevant and important, although there are clearly more:

- the structural relation between language and mathematics
- the registers and discourses relating to mathematics
- the interactions in the classroom
- the different theoretical tools and approaches (eg. linguistic approach, Vygotsky social/cultural approach, education didactic approach).

The interplay between such broad descriptors as these may well be a framework for generating useful research questions.

In developing important research questions, one avenue that should perhaps be taken far more seriously is what teachers are saying. It has been noted above the changing and complex role they are asked to live out in mathematics classrooms. What teachers have to contend with in their day to day teaching experiences may not readily match the theoretical thinking and rhetoric expounded on at various conferences and in journals. This could lead to a gap between accepted theoretical knowledge and teacher knowledge. Such a gap can give rise to potential dilemmas, but these in turn can lead to insightful questions. To this end, we need good practical descriptions of teaching within multicultural classrooms, which may be best generated by teachers. This would give researchers the classroom context as seen by teachers in order to inform the research questions developed perhaps by teachers in consultation with researchers. In other words, the culture of the practice of teaching should be a rich resource for research questions and may well lead to possible ways forward in our theorisation as well in our attempts to help generate more insightful practice. It is probable in this dialog that the researchers' perspective with its wide ranging resources and knowledge of theory may well give a general frame for such teacher generated questions. Hence a dialogue between the two is needed, as both teachers and researchers stand in the overlap of their domains. The newly announced ICME study may well contribute to this dialogue (Ball & Even, 2004).

Finally, we need to guard against not theorising within mathematics education, but only using other perspectives to look in on our own context. The responsibility for theorising our own field rests with us. Such an approach should not prevent us using a technique or concept from another field, perhaps as a start for our own new thinking. The question becomes how to use these other perspectives, without using just the surface features of the theory only. Another aspect of this issue is how to properly appreciate the depth of the 'other' approaches. We need to be explicit about what theories we are using and how we are using them. We are involved in the creation and recreation of ideas. There will always be a tension between using others' ideas, and understanding the original reference framework of those starting ideas. Therefore, we need to spell out the way we are using an idea, how it is to be understood in our reference frame, and just as importantly how it is not to be understood within our reference frame.

This paper has underlined the fact that many classrooms in which mathematics is taught are micro sites of multiculturalism. With the recognition that mathematics itself, and more clearly what and how mathematics is taught, is influence by culture, language and the social milieu of the classroom, school and the wider society, deeper and complex issues for research immediately become the foreground. There are implications with such recognition for some traditional markers of what makes a successful student and/or program. For example assessment may no longer be considered the only marker of success. However, an analysis of these issues shows that there are differing contexts that may be important in such research. In this paper the different contexts and situations that arise with language have been briefly explored, but the same can be also done for culture and other influences. There was no attempt here at deeply analysing the implications of such complexity, suffice to say this may be important. The final comment in the paper suggests that when analysing these implications, the role of the teacher in developing good research questions should not be overlooked. As well, due acknowledgement to the source of theory building in this area should be always given, as it should in all of mathematics education research.

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[1] This paper draws on the paper written as an introduction to the collection of papers given during the meeting of the Thematic Working Group 10 ‘Teaching and learning mathematics in multicultural classrooms’ at the third Conference of European Research in Mathematics Education, held at Bellaria, Italy in 2003.

The reference to that paper is:

Gorgorió, N., Barton, B., & Clarkson, P.C. (2003). Teaching and learning mathematics in multicultural classrooms. <http://dlibrary.acu.edu.au/maths_educ/conferences.htm>

The full collection of papers for the Working Group are found at: <http://dlibrary.acu.edu.au/maths_educ/conferences.htm>

Acknowledgement is given here to a number of ideas used in this paper that were first formulated in the discussion of the Working Group.

Readers are encouraged to consult the original papers for further insights.

NOTES ON TEACHER EDUCATION: AN ETHNOMATHEMATICAL PERSPECTIVE

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Why the term “notes” in the title of this paper? Surely, I cannot answer such a question accurately, but its use in this paper intends to guarantee, from the start, that the ideas discussed here are being developed and have not reached yet the desired degree of elaboration for such a special responsibility. Nevertheless, leading such discussion might help to sort out the ideas on our object of study – teacher education - and to deepen it through an ethnomathematical perspective.

From some years teacher education has been thought as a key issue towards more effective transformations in the school system; it has not only been thought in reference to the instruction but also regarding the construction of values on the road of school education. By one side, for some time now, most of our educators, through different processes, realize that most part of the ideas/conceptions, from the time we were educated, are now meaningless and do not satisfy in order to develop studies on didactic-pedagogic issues. By the other side, the elaboration of alternative propositions - suggested to teacher education – are in the sense that the teachers ponder over their practices and share their decisions. In fact, the teachers have been invited to have their own opinions and to express them, as well as to participate more actively in the educational political-pedagogic project as a whole.

Since we are talking about education in an ethnomathematical perspective, I would like to begin this discussion describing some few moments of teacher Mário’s class - not only to illustrate this paper with his attitude, and then, perhaps, expose it to a favorable critique, but also to apprehend part of a school reality that could serve, alongside this work, as a significant example to enlighten, by means of comparison, what I will be proposing. Mário is a middle and high school mathematics teacher at a public school in a São Paulo suburban county, to whom I requested, during our sessions discussing teaching, to begin the mathematics class, whenever possible, with the students’ speech, starting with questions like “What do you know about...?” or

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"How do you understand...?". Mario and I used to talk, for instance, about how we, teachers, needed to review our attitude towards the knowledge we have of how the students know and how to work with "this knowledge" regarding the scholar knowledge. Below there is a passage extracted from the mentioned class:

> *Mário starts, in one of his fifth grade class, to chat on division calculation with the students, by asking:*
>
> **Teacher Mário**: How do you calculate 125 divided by 8?
>
> *José, who sells chewing gums at a traffic light downtown, begins to speak:*
>
> **José**: We are about 10 "guys", almost every day, some boys and some girls. Then, we divide like this: more for the girls that are more responsible than the boys, more for the biggest than for the smallest ones.
>
> **Teacher**: Give me an example José. For instance, how was the division yesterday or the day before yesterday.
>
> **José**: Ah! Like this... there were 4 girls, 1 is one of the smallest; 6 big boys and 2 more or less small ones. Then we were 12 and we had 60 chewing gums. Then, it was given half and half, a little more for the girls. The small girl got 3 and the other ones got 6 or 7, I don't remember well ... The boys...
>
> *Then prof. Mário invites the class to divide other amounts of chewing gums and other numbers of boys/girls using José's group division method ...*

Which types of teacher education discussions/practices, could have sensitized/influenced teacher Mário? Recognizing that every process of teacher education is built in a way of strong ideological concentration, which values incorporated by the teacher Mário might have influenced his practice? Still about Mário's professional education, concerning the pedagogic actions he has been presented, what is more outstanding: the contents? The ultimate purposes/goals of the curriculum? the students' learning processes? The students' cultural/social aspects that may positively or negatively interfere/intervene in the academic performance? The preoccupation of taking into account the students' cultural aspects in their education?

In an attempt to recognize which teacher education perspectives might have made Mário much more aware of the role of social aspects in education, I will discuss some conceptions/propositions that, for the past years, have been guiding teachers' education. However, I want to make it clear, from the start, that my tendency when directing this questions is to display if educators that have discussed issues on this topic have distinguished, among others, two points: first, it is not possible to develop somebody aside from all of his/hers social-emotional-cultural experience of life and, second, students are not alike.

FIRST, IT IS NOT POSSIBLE TO DEVELOP SOMEBODY ASIDE FROM ALL OF HIS/HERS SOCIAL-EMOTIONAL-CULTURAL EXPERIENCE OF LIFE AND, SECOND, STUDENTS ARE NOT ALIKE.

Even though this being a discussion on ethomathematics issues, I do not intend to present, exhaustingly, the presuppositions and issues of this area of studies. In fact, I will not go to depths on that proposal, but I will merely and briefly speak about the role of the one social group's knowledge in relation to the “other” one, I mean, what the "other" knows and the value that would be attributed to this knowledge.

Any way, Ethnomathematics as a line of study and research of mathematics education, studies the cultural roots of mathematical ideas that is given by ethnic, social and professional groups; in other words, the ethnomathematics studies, attempting to follow the anthropological studies, try to identify mathematical problems starting from the “knowledge of the other”, in their own rationality and terms. D´Ambrosio´s different interpretations, in different moments for the past 15 years, can lead us to a better understanding of this subject:

> ethnomathematics reveals all mathematical practices of day-to-day life, or preliterate cultures, of professional practitioners, of workers…includes the so-called academic or school mathematics, taking into account their historical evolution, with the recognition of all natural social and cultural factors that shaped their development…different forms of doing mathematics or different practices of a mathematical nature or different mathematical ideas or even better mathematical practices of a different form… the art of explaining mathematics in different contexts…so many people, so many mathematics…different forms of people mathematize… this line of research searches the roots of mathematics – it searches the history of mathematics.

Usually, in the scope of ethnomathematics research, the researcher lives a process of strangeness and tension, since the quantitative/special relations noticed inside the investigated group – even if he is not exclusively centered in the explanations of his/her society – reveals to him or her to be illogical and, in general, a process of their re-significance and analysis asks for the creation of categories that would involve articulations between mathematics and several other areas of knowledge, such as history, myths, economics. In other words, the creation of categories in a transdisciplinary dimension it would be necessary. This, of course, is a big challenge, and in most part of the studies, the ethnomathematician interprets though the concepts of "his/her" mathematics.

When the focus of the study is the pedagogy of mathematics, both have been the challenges:

- legitimizing the students' knowledge grown from experiences built in their own ways and;
- the possibilities of how to work from this knowledge, I mean, from the ones outside the school and the ones inside the school.

Indeed, the aim of ethnomathematical studies is to help the teacher establish cultural models of beliefs, thought and behavior, in the sense of contemplating not only the potential of the pedagogic work that takes into account the "knowledge" of the students, but also a learning inside the school, more meaningful and empowering.

Perspectives of the teacher education

Several models have been proposed regarding teacher education – most of them has been dealt with the teacher as a social subject of his/hers actions and are centered on the formative dynamics of these processes of transformation.

Indeed, in the last years, the discussion around teacher education has, on the one hand, left in second plan the teacher education towards the teaching of the contents of a specific area; on the other hand, it has been stressed the importance of the teacher as a reflexive professional. Being reflexive has been discussed in the sense of exercising the teacher's reflexion on his/her interactions with the needs of the students.

Naturally, to re-think the pedagogical actions involves to ask the teachers to pay attention and comprehend, in a more appropriate way, the student they receive, that is ask to them to look at the question "Who are my students?". Looking at this perspective, I can recognize, beforehand, that the students have not been completely outside the proposals of teacher education, but they are not inside either.

THE STUDENT HAS NOT BEEN COMPLETELY OUTSIDE THE PROPOSALS OF TEACHER EDUCATION, BUT THEY ARE NOT INSIDE EITHER.

One line of research, on teacher education studies, that has been guiding the most current discussions, is the one of *reflexive teacher*. Since the 80's, the original ideas from DONALD SCHÖN, have been stressing ways of operating of *reflection in the action* and of *reflection about the action*. According to the author, these are two important attitudes of the competent professionals/educators and it is from the reflection on one's own practice that transformations can happen.

The movement of the *reflexive practice* appeared in opposition to the idea that the teacher is a transmitter of a number of pre-established information and it started to guide, worldwide, the teacher education specialists' discussions, as we can see in GARCIA (1997), SCHÖN (1997), ZEICHNER (1993), NÓVOA (1997), CARVALHO & GARRIDO (1996), JIMÉNEZ (1995), FIORENTINI (1998), among others. Generally speaking, the conceptions that steer the reflexive formation of the teachers emphasize that *teacher education* should have, as its main goal, the teacher reflexive self-development (NÓVOA, 1997); in other words, to form teachers that learn how to cope and understand not only the intellectual problems of the school pedagogy as well as those that involve the reasoning of each student.

From the point of view of our discussion about teacher education, in an ethnomathematical perspective, some initiatives inside the reflexive formation have been precious, especially the one named "giving reason to the student" (SCHÖN, 1992), that stresses the teacher investigating the reasons behind certain things the students say. On one hand, the idea of teaching and knowledge, through the teacher that *agrees with the student*, indicates that *the student's knowledge must be within the formation proposals.* Somehow it has been emphasized that the teacher should recognize and value the student's intuitive experimental daily knowledge, for instance, s/he tries to understand “how a student knows how to change money, but doesn't know how to add the numbers” (SHÖN, 1992) or how the candy seller students perform the division, not a division in equal parts, but a distribution based on social-emotional reasons (our teacher Mário!). On the other hand, the teacher education specialists, via *reflexive teacher,* have still a lot to learn with specialists of some specific areas such as anthropologists/sociologists/historians, among others, in order to learn from them that the student's development, although in the school context, is a phenomenon of holistic proportions. It must been interacting, in the school context, the emotional, the affective, the social, the historic, the mystic, the cultural, among others aspects. Actually, our search is linked to the fact that "the Ethnomathematics is situated in a transition area between the cultural anthropology and the mathematics we call academically institutionalized, and its study open to the way to what we could call *anthropological mathematics*" (D'AMBROSIO, 1990).

Surely, there are many other specialists involved with the models/methods/foundations of teacher education characterization, such as PONTE (1994, 1999), SHULMAN (1986), COONEY, (1994) and more than in the past, the representatives of this line of research have been tried to take into account the cultural and social aspects that can intervene, positively and negatively, in the student's academic performance as well as the values and the purposes of this attitude. Naturally, the last observation reveals growth in terms of research and researchers with the possibility of a joint construction, with understanding and articulated towards this direction.

Still discussing the concern of some educators, in teacher education- with the student's knowledge, his/hers interests and learning processes, it is important to point out that D'AMBROSIO who has emphasized some characteristics to be incorporated by the mathematics teachers facing the current curriculum reforms. Indeed, she points out some questions about helping our students to establish a positive relationship with mathematics (D'AMBROSIO, 1996) and, to accomplish this, values the attention for the first knowledge of the student. D'AMBROSIO quotes:

> The main ingredient of the electric outlet of the teacher's decision with relationship to the direction of the classes and of the student's learning it is the discovery, for the teacher, of the student's knowledge. The student arrives to the educational process with a wealth of experiences. The mathematics teaching (and, in fact, of most of the school disciplines) not more it is based in the structure of the discipline, but on the contrary, it is based in the student's knowledge. For so much the teacher needs to organize the work in the room of way class in order to elicit the student's knowledge so that this knowledge can be analyzed.

> It is also important to create activities that take the student to look for in your experiences knowledge already formed.

Indeed, we tried to consider up to this point the need of taking into account, in the space of the discussions on teacher education and mathematics learning-teaching, the first knowledge of the student linked to the cultural model to which s/he belongs - a perspective that is opposed to the tendency of the so-called traditional school, of treating the students as if they are all alike. Actually, giving to the student a neutral value as well as the use of the same methods and contents to all–also an old posture of the so-called traditional education–has the universal pattern in terms of teaching, that is, the teacher as representative of a group that detains the knowledge is the one that can offer to the student an option to go from the common sense to the understanding of the science (D'AMBROSIO, 1990).

In fact, the critic in the sense that *the school treats all the students alike has been out* there for a long time and is, in general, a reflection of social-political-economical order, linked to the issues of education and power, education and ideology and education and culture. Among others, NIDELCOFF (1978) calls clearly for our attention, in this sense, for the political-social meaning and consequences of this attitude:

> The school will treat all for equal. However they ARE NOT SAME. In function of that, for some so many ones it will be enough that the school gives them; for others no. Some will triumph, others will fail. That victory will confirm those to who the society supplied means to triumph. And the failure will usually confirm the contempt to those that the society conditioned as inferior.

A more systematic elaboration of this discussion has been found in *theory of the social reproduction*, in the sense that the school is a mechanism of reproduction of the dominant ideology and of the addictions of the dominant classes. BOURDIER and PASSERON are quite active representatives of this line of thought in Europe, with reflections inspired in DURKHEIM, ALTHUSSER and GRAMSCI whose studies have been systematized Marxism's new theoretical approaches. A similar movement also happened in the United States, inside a line called Political Economy of the Education, with CARNOY, APPLE, GIROUX, TORRES, among others. APPLE and GIROUX developed/complemented the reproductivist thought from a cultural perspective, pointing out, for instance, that the students come at school distinguished in social classes and leave school also distinguished in social classes, because all curricular outline is a construction free from values, neutral, elaborated impeccably and discussed with and among teachers/educators also involved in curriculum development who, in general, don't intend to get into a social debate about such elaboration. Anyway, the investigations on teacher education that take into account the theory of the social reproduction are rare. It seems that everything happens as if great part of the educators was attentive and in agreement with this vision, but its configuration could not get into directly in the cognitive orientations and in the identification of these discussions. In this perspective, two questions remain, whose answers could, perhaps, go deepen into the issues on teacher education, in a Ethomathematics perspective. They are: does the social power have the power to transform the affective-intellectuals relationships with the political authority of one group? Can the teacher education, while a cultural practice, transform/reduce the segregationist education function?

In an attempt to answer the questions as well as to locate, in the Brazilian history, a teacher education project that has the student as its central focus – especially, the attitude of taking into account the student's previous knowledge - I will take, as foundation, PAULO FREIRE'S literacy that "being a political act, as all education, is a act of knowledge" (FREIRE, 1980: 139). Indeed, this FREIRE's statement is a result of his conviction that "in every relationship between educator and student it is always at stake something that one tries to know" (FREIRE, 1980: 139).

As it is well known, FREIRE developed and ran through such method as an option to reveal the extreme link/coherence between political and practical educational action. I do not expect here to speak of the method in itself–since its importance is very known–but merely and briefly, to refer to some of its aspects, as the concern in bringing the teacher to take as reference for learning the reality of the people and the concern in seeing such reality referred in the "generating words" and represented in the "code" that is analyzed and discussed with this people (FREIRE, 1980: 140).

Actually, FREIRE's proposal of bringing the teacher to turn to his/hers students is fundamentally different from all the pedagogic positions and precedent epistemologies. Such statement is justified at least for the author's two attitudes/positions: first, according to FREIRE, the role of the teacher in the group is not the one who tries to interact with the student discussing the relations between specific contents and even less it is not the one who transmits knowledge, but the one who, through the dialogue, tries to know along to the students–when teaching something to the illiterate the teacher also learns something from them (FREIRE, 1980 P.140). In fact, FREIRE places the educational action in the student's culture. For him, the consideration and the respect of the student's previous experiences and the culture that each one of them brings inside themselves, are the goals of a teacher that sees education under the libertary optics. In other words, he recognizes it as a way to generate a structural change in an oppressive society–although he recognizes that it doesn't reach that aim immediately and, even less, by itself.

The Freirean proposal, therefore, for teacher education, from the point of view of the contents is to bring the teacher to highlight the programmatic contents from the investigation of a significative thematic to the student and to dialog with the student about his/hers vision of the world on such themes-which reveal themselves in several forms of action-and the teacher's. FREIRE believes:

> it is necessary that educators and politicians are capable of knowing the structural conditions in which people's thinking and language are constituted dialectically… the programmatic content for action, that belongs to both peoples, cannot be exclusively elected from them, but from them and the people… It is in the mediatory reality, which the conscience of educators and people, are going to look for the pragmatic content of education… The moment of this search is what inaugurates the dialogue of education as a practice of liberty. This is the moment that the investigation of what we call the thematic universe of the people or the collection of their generating themes is realized.

It is worth here to <u>detach</u> that FREIRE looks at teacher education by the side that some <u>distrust</u>, the one of the space for the oppressed to make their accusations/denunciations/complains. However, from my point of view, what he discussed from the decades of 1960 and 1990 was of absolute importance to the teacher's education in an ethnomathematical perspective. Which perspective is this? The answer to this question is an enormous challenge–as it is well shown by the whole plot of this text–that <u>fits</u> to the teacher to answer in their practice. Certainty, there is no recipe. But, surely, a good example is in teacher Mário's performance that seems to deal with the presupposition that someone's knowledge, about something, is never neutral and does not happen as if it was a hermetic event, in a specific moment. On the contrary, every student, adult or child, has a conception of one aspect of knowledge that results in his/hers history of learning and it is that knowledge, in the condition it is found, that will make the filter between s/he and the new knowledge. This can be better understood in the context of Mário's class: if the student has a conception of division as proportional parts in terms of his/her gender and age friends, when s/he hears about division s/he may not consider it in equal parts, as it is in formal mathematics...

In conclusion, it is worth to highlight that, on one hand, the possibility of such attitudes on the part of the teacher–who try to negotiate with the universe of the student's knowledge and, in doing so, can be less authoritarian and more dialogic - they are intimately linked to the way of being of the teacher as a human being, in the daily life, as well as to the knowledge the teacher has of him/herself and of the school context. On the other hand, in regard to the pedagogy via Ethnomathematics, it is natural to think teacher education not only returned towards a new vision of Mathematics and its appropriation by the students, but also towards the scientific and pedagogic general updating of the mathematics that is out there, in such a way to contest or to incorporate it, as much as possible, in the situation-problem in question.

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QUANTITATIVE AND SPATIAL REPRESENTATIONS AMONG WORKING CLASS ADULTS FROM RIO DE JANEIRO

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This paper reports some of the results of an ethnomathematics research developed with a group of low educated adults, living in a poor neighbourhood of Rio de Janeiro. The research aimed to understand quantitative and spatial representations built and used in different life contexts, as well as relationships between these representations and school mathematical knowledge. Results showed a strong association between the use of mathematical skills in daily life and survival strategies to satisfy basic needs such as managing a reduced budget, but appeared to be related also to emotional factors like protecting one's identity. In contrast to exclusively cultural factors, the research results make clear the predominance of social and economic factors interfering in the building, representing, and using of mathematical knowledge in an urban context.

Keywords: ethnomathematics, adult basic education, quantitative and spatial representations.

Research presentation

In a city like Rio de Janeiro, many of its actual working class population is originally from other poorer states of the country that migrated many years ago and settled down in neighbourhoods with underprivileged survival conditions, the so-called *favelas.* Among them, there is a significant percentage of adults that could not complete in the past the first years of basic education. Many of these adults now look forward to improve their school education, and be able to get better paid jobs. However, regular school is not prepared to deal with this kind of student, that is undereducated but beyond school age, looking at them from a negative perspective[1], for what they do not

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[1] As stated by Oliveira (1999).

have (school degrees), what they are not (children), and what they do not know (school knowledge).

Based on previous studies (Carraher, Carraher & Schliemann, 1988; Abreu, 1993), we were aware of contradictions between adult student's success with their own ways of reasoning mathematically and difficulties they experience with school mathematics. Thus, my doctoral research (Fantinato, 2003) aimed to view this population from a positive perspective, trying to understand their daily mathematical knowledge, related to school knowledge they experience when they return to basic formal education. My initial research questions were the following:

- What kinds of mathematical knowledge do working class adult students build in different life contexts?
- How can out-of-school mathematical knowledge be related to school mathematical knowledge?
- Can a better understanding of different kinds of mathematical knowledge, produced by low educated adults, be useful to educational practices with similar students?

Literature review on adult mathematics education studies was proceeded, in order to have an overview about their contribution to the questions above, and three categories were created, according to their main goals. Adult mathematics education could be seen as a means of *developing political conscience* (Duarte, 1985; Knijnik, 1996), could be mostly related to the *development of labor market competencies* for a highly technological society (Gal, 2000; Singh, 2002) or could be in search of *adults own ways of mathematical reasoning* (Carvalho, 1995; Toledo, 1997; Harris, 2000). My work, studying processes of mathematical reasoning among a group of adult students from a social-cultural perspective, can be positioned in the third category.

The theoretical framework of this study was based on *ethnomathematics*, since it concerns the social and cultural roots of mathematical knowledge of specific groups. The acknowledgement of students' particular strategies that diverge from school mathematics has a strong potential for their *empowerment*, which is related to the *political dimension* of ethnomathematics, underscored by D'Ambrosio (2001) as the most important one. An ethno-mathematical approach would be also the most suitable to study out-of school mathematical knowledge[2], particularly with people that had already been excluded from school system.

Therefore, an ethnographic research method was developed during the year 2000 mostly, in a poor neighbourhood of the city of Rio de Janeiro, the *Morro de São Carlos*. Routines of a local course for adult education were monitored, as well as of aspects of the daily life of the students and their community. The following research techniques were employed: participant observation, interviews, documents analysis and photography.

The project had thirty students aged between 19–75 years old, divided in two groups, according to degrees of literacy and years of elementary school in the past. Students worked in the following activities: domestic maids, sellers, sewers, construction workers, waiters, retired, polisher, cooker, doorman, or were unemployed. Most students came from other Brazilian states, especially ones in the Northeast. In spite of cultural differences due to geographic background, religion and professional experience, they shared the common characteristics of studying in the local course and residing in São Carlos, constituting thus a *social group*.

A dialogic posture between researcher and subjects proved to be a crucial way of allowing the researcher to approach the research universe, by making the familiar strange and making familiar what was strange (Da Matta, 1978), in order to understand explicit and non explicit meanings of the group. Another methodology approach that I employed was to constantly acquaint myself with the research subjects, trying to grasp different points of view of the same aspect of reality. This also provided a form of triangulation.

Data collection and analyses first focused on quantitative and spatial representations of aspects of the quotidian life in the community, then focused on the building, representing, and using processes of mathematical knowledge by adult students, in school and outside school contexts. This paper's goal is to introduce some of the results found, that could provide a glimpse into "knowledges, techniques and practices" (D'Olne Campos, 1995) that could be related to mathematics, by an urban group of low educated adults, living in poor neighbourhood of Rio de Janeiro downtown.

[2] As stated by Lindenskov &Wedege (2001:12): "By taking an interest solely in the praxis within the formal system of education, it is difficult to see that personal intentions, media, and situational contexts emerge as being equally important to skills and understanding."

Getting to São Carlos: quantitative and spatial relations in community's means of transportation

The Morro de São Carlos is a little more than seventy years old, and is one of the oldest *favelas* in Rio. In the past, people used to climb up the hill by foot, because streets were not paved. Nowadays, one can use local means of transportation, such as *vans* and motorcycles, or even go by car, up to a certain part of the community. Some narrow lanes are only reachable by foot.

I could perceive differences between local habits and general city rules right in the beginning of field research, when I used to take the vans to go up the hill. There was no time regularity for those vehicles' departure: the conductor waited until there were a minimum number of passengers. Depending on the hour of the day, it could take long to fill the car with what seemed to be *enough*, usually at least 9-10 persons. Apparently, these rules became more flexible when going down the hill: the van would go with less people and even the ticket fare was cheaper.

What seemed to be basically an economic criterion later could be associated to a different and more social one. In the vans system, children under eight years old do not pay but may not be sited: they had to stand or sit in parent's lap. People of São Carlos, even the children, seemed to have understood this particular code, and creatively dealt with it, as shown by the situation below, taken from my field notes:

> The van goes down the hill empty. The conductor makes a stop and a little boy goes in. Initially he stands up, and then takes a seat in front of me. Starts chatting with me. I ask how old he is. He answers in a low tone of voice, while showing an ad that says that children over eight must pay the ticket: "I am eight, but I look half my age, because I am tiny". (06/09/00)

The boy's attitude, sitting only after checking that there was no adult to be seated, and *hiding* his age, is an example of how social norms create particular rules, in this particular world. Probably the vans' owners accept to be flexible in this case since they know community's life condition, where to save ninety cents (ticket's fare) would be relevant for family budget.

In the context of this local transportation system, another instance of the complexity of quantitative representations is the fact that the vehicle would leave around rush hour, on a rainy day, with seventeen passengers, among them paying adults and children under eight. In this case, *ten* (the car's limit) could be represented as *seventeen*, which led to the interpretation that in São

Carlos daily life, quantitative and spatial representations seemed to be more related to social demands than to safety rules.

Geographic conditions, as well, influence the spatial representations of São Carlos' inhabitants. During my first attempt to go to São Carlos with my own car, while driving up a very steep street with a sharp curve, I nearly hit another vehicle that suddenly appeared in front of me, coming on the left lane. My surprise was increased with the observation that in that particular curve, all vehicles would make an inversion of the official traffic rules[3], and also that I could not see any notice, warning drivers of this change of course. Later I found out that this unusual spatial representation was familiar not only to São Carlos's inhabitants but also to its frequent visitors. However, *foreigners*, like taxi drivers, had to be told of the so-called *mão inglesa* (English direction) before causing accidents.

I could make different interpretations of this change of direction in this street of São Carlos and of inhabitants' attitude towards it. One of them is related to geographic conditions. When driving up the hill, the car loses power because of the steepness of the street. Since in that place there is a sharp curve, it's easier to drive pulling the car to the left, and keeping speed stable. From an outsider point of view, this change of directions could cause accidents. But this spatial representation by the residents of São Carlos seems more connected to the physics of the situation and local history[4] than to official traffic rules.

Secondly, the community's carelessness about the need to clearly call drivers attention to the change of direction in the curve, seems to be a way of representing their own world as separated from *outside-favela* world[5].

[3] In Brazil, like in several countries, cars must drive on the right side of the street.

[4] Going down the hill, there are two streets that run into only one, what makes the driving even harder in that curve. This change of direction seems to have been stablished from an upside point of view, what is compatible with the way *favelas* grow, with no architec planning for them. Streets were opened and paved with the help of inhabitants, that used to walk, for there was no other way to reach their residences.

[5] Another example of differences between the *city* world and the *favela* world can be found in the way houses are numerated. While in the whole city buildings are numbered according to an official rule, odd numbers in one side, even numbers on the opposite side, following an increasing order, in São Carlos one can easily find in a street two houses with the same number, or a house of number *7* between one of number *10* and another one of number *16.* This can be explained by how community grew, by people building houses one after the other on empty spaces, and choosing which number to put by looking at next door neighbour. Those house numbers seem to be more like *names* than numeric references.

A third point is a methodological reflection about in which situation I perceived the difference between this rule and my own spatial representations. In spite of having been to São Carlos many times before (taking the van), I could only see the difference when driving my own car, that is, from my own cultural point of view. Dialogue with the *other*s' culture, thus, seems to be only possible when people recognize each other within their differences.

Adults' mathematical strategies at the food-store or: *overestimating*[6] *not to be ashamed at the cashier*

When asked about where did they use mathematics in everyday activities, subjects' immediate and quick answers were similar, and refer to the act of going to the store and buying food supplies. These are some of their words:

> If something costs two reais[7] and eighty cents, I say, it's three reais. I say so…to know if my money will be enough to pay! If something is one real and eighty, I say: two reais. If it's five and forty, I place six reais. I do that way because I can pay for it and I know *I will not be ashamed when I get to the cashier.* (I 7[8], 28/09/00)

> The money you get...you don't get enough...you go the supermarket...if you don't carry a pen...making notes...getting from the supermarket and making notes with the pen...if you are going to buy a little something...you *might feel ashamed at the cashier*! Because you have little money, and you keep getting things, filling the cart out... (I 5, 28/09/00)

> I keep writing prices all the time, but always rounding off,…*for not having to face the situation of not giving the money when I get to the cashier…* (I 3, 15/09/00)

Many issues arose from the above statements. In the first place, the rounding up procedures, with mental calculation or written records, comes from the need to estimate the amount purchased before paying for it, in a domestic organization without checks or credit cards. People have to spend the exact amount, or little less than they carry, saving some cash, for instance, for transportation back home. The procedure they adopt is to round every merchandise's price up the next whole number, apparently *disregarding* the

[6] From the Portuguese « calculando exagerado ».
[7] *Real*, and *reais* (plural) are the names for Brazilian money.
[8] I 7 – code for Interview 7.

cents, as the first student said: “if it’s five and forty, I place six reais”. Mathematics precision, in this case, is not as important as creating survival strategies.

Nevertheless, there is another reason for the mathematical thinking in the statements above that seems to relate to emotional factors. The overestimation is done to prevent adults to face the embarrassing situation of not having enough money to pay, when they get to cashier. All research subjects were aware of this overestimating need, and expressed the same feeling about avoiding *being ashamed at the cashier.*

Why would adults from São Carlos, in particular, have this unexpected motivation for mathematical reasoning? I could understand it better looking at context. Individuals living in a *favela*, where borders between *drug dealers*, *thieves,* and *honest people* are not easily perceived by the outside society, means having to live daily with a negative social representation of their community. This reality leads people to create strategies to protect their self-esteem[9]. Besides their lack of school education, research subjects social marginalisation is increased, since they belong to a low-income social class.[10]

Another kind of mathematical thinking in the food-store context, comes from the act of making choices among goods to buy. In the subject’s words below, he is clear about his priorities on spending family budget, by using some kind of ordination reasoning:

> Meat is what costs more…food is cheaper than one kilo of beef…If you pay eight and forty on a meat kilo, look how different it is…One kilo of rice I think it’s one real and fourteen cents…It’s too big of a difference…My opinion is to first get the main part, and only later think about meat. If there is any money left for meat, OK, but if there isn’t, the main part is safe. (I 9, 07/10/00).

Data analysis also revealed that subjects employed a creative way of comparing numbers representations by estimating prices. In the case below, a sewer’s practical experience helped her mathematical reading:

[9] One example is local habit of changing one’s address when filling employment files. Another is placed by post office, that considers São Carlos as a *risk zone*, and sends much of inhabitants’ mailing post to community association.

[10] Garcia (1985) also approaches this feeling of *shame* among low-literate adults, stating that they interject a guilt feeling for not having literacy skills, in a urban and increasingly literate society.

> I feel ashamed, sometimes when I come to a store to buy clothes, I choose by texture, because I see those numbers and I don't understand…If it is made with a good cloth, I know that it is expensive…So this is not ten, this one hundred…In order to understand mathematics, I go by the quality of things…(I 3, 15/09/00)

Lave, Murtaugh & De la Rocha (1984) also studied arithmetic procedures in the grocery shopping within a group of middle-class small town inhabitants in the U.S.A. They found that sometimes qualitative criteria, such as preference for some brand, or the storage space at home, could be more important for decision making about what to buy than the list of merchandises on sale, arithmetic procedures being not necessary in those situations. Comparing Lave's subjects our own, we could state that among adults of São Carlos number calculation and estimation were constantly present in the context of supermarket. Since middle-class Lave's group belonged to a wealthier social class than the adults from São Carlos, differences found between the two groups emphasize the importance of the social, economic and cultural context influence on cognition processes.

Conclusion

Among low educated adults from São Carlos, quantitative and spatial representations seem to be determined by a double necessity. First, *to survive*, calculating, estimating and comparing being used to satisfy basic needs, by managing a reduced budget. Secondly, to preserve one's *identity*, as an individual and as a member of a community that bears a triple stigma of exclusion: inhabitants of a *favela*, members of a low-income working-class, with underachieved school education.

This research has stressed the predominance of social and economic factors influencing the building, representing, and using of mathematical knowledge in an urban context, making evident these features are a significant identity factor, going beyond exclusively cultural factors. Therefore, the research results and the consequent issues that it raises, might contribute to discussions of the restricted concept of ethnomathematics as the study of mathematics of a specific cultural group, to a broader one. As Barton (2002: 2-3) states:

> There are ethomathematicians who work within their own culture, however the ethnomathematical part of their work is the interpretation of their own culture (or of parts they wish to call mathematical) in a very way which is understandable to those outside the culture. Such activity is still

dependent in a theoretical way on some concept of mathematics – a concept that, in its international sense, is not internal to any one culture.

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INTERCULTURAL MATHEMATICS EDUCATION: COMMENTS ABOUT A DIDACTIC PROPOSAL

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The practical implementation of the theories developed in the area of ethnomathematics research and culturally contextualized mathematics education does not seem to have been dedicated much attention in some countries where multiculturalism is a relatively recent educative requirement. This article presents some considerations made by mathematics teachers and their pupils after completing an experimental intercultural and interdisciplinary didactic proposal elaborated in Italy in the context of the IDMAMIM European project. The proposal will be presented in more detail during DG15 activities.

Education in multicultural contexts

An ever-increasing multiculturalism represents one of the most significant new features of society today. In some cases, the countries involved in this phenomenon have demonstrated to be inadequately prepared to confront it either from a legislative or social point of view. In particular, although the scholastic system has taken note of the changing socio-cultural context in the classroom, it has not always been able to make educative choices which would take this change into consideration: this lack is most apparent in the area of disciplines teaching.

In fact, although praiseworthy attention has been focused in some countries on the in-service training of teachers covering the general themes of multiculturalism and intercultural education, little or none has been given to teaching methods and paths to be used in this new class context that is being formed and is constantly under transformation.

A little attention has been given (and it couldn't be otherwise!) to teaching the local language as L2 language; but even in this case, until recently, attention has been focused on the language itself, with little or no attention given to the special language that each scholastic discipline requires, words that have developed independently in each language and that are sometimes

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even found to have conflicting meanings (see [Favilli and Jama] for the Somali language and mathematics). It has only been recently that special language has provoked the interest of experts active in the sector of teaching L2 (see [Favilli, 2004b] for the Italian language and the scientific area). To be truthful, some attention has been focused at a research level in some disciplines such as mathematics, in terms of teaching in multi-language class contexts, but there is very little or no impact of this on the school environment. It must be said then that up until now, attention has been shown above all with reference to English speaking educational contexts.

Countries, which have not yet confronted the subject of intercultural teaching of disciplines, include those where the phenomenon of the presence of cultural minority pupils is more recent: this group certainly includes of southern Europe countries.

Another significant aspect of today's schools is the widespread understanding that the teacher’s educative role is becoming ever greater. Society entrusts the scholastic system with ever wider and more responsible tasks: teachers are not simply asked to teach, but to involve themselves in activities that educate in how to live healthily, road safety, intercultural education, etc.

As a consequence, teachers have found themselves in recent years having to rethink the content and structure of their curricula and to change their way of teaching. Pedagogues clearly indicate the fundamental importance of teachers placing the pupil at the centre of educative activities. Didactic programs and methods must take the class context into consideration, and not simply consider it as a undifferentiated unit. The class must be considered as a group of individual pupils, each with their own personal and family history and with their own culture.

Mathematical education in multicultural contexts: the IDMAMIM[1] project

In this new socio-pedagogical scene, teachers also have to ask themselves the question:

> *How can I teach mathematics while taking into account the personal experiences and cultural background of the single pupils present in my class?*

Whatever the response to the question might be, the teacher is obliged to reflect on his or her certainties, some of which may be transformed into doubts , leading the teacher to rethink some of his or her beliefs. The main belief is without a doubt:

> *Mathematics is universal (a universal language) because it isn't conditioned by different cultures.*

But this is in fact the first belief to be shaken. And how could it be otherwise! Classes of many schools are now increasingly characterised by the presence of some pupils originating from countries with different cultures or pupils from cultural minorities: a teacher aware of his or her educative role easily recognises that each of these pupils brings an extraordinarily different cultural baggage both in terms of content and form. From this knowledge, the teacher can certainly identify mathematical type knowledge; knowledge that nearly always lead back to mathematical activities that are typical and characteristic of cultures different to the majority western culture. These mathematical activities have not yet found any trace in the mathematical programs or texts used in our schools.

Well, an attentive teacher must therefore admit that mathematics, as a result of various mathematical activities, isn't and can't be universal, since mathematical activity, as a product of man, is conditioned by the culture and the society of the place where it is performed [Bishop].

[1] IDMAMIM – *Innovazione Didattica MAtematica e sussidi tecnologici in contesti Multiculturali, con alunni Immigrati e Minoranze* – (innovation in mathematics didactics and technological aids for multicultural contexts with immigrant and minority pupils) a three year trans-national project (2000-2003), developed under the SOCRATES-COMENIUS program, action 3.1 (training of teaching personnel), overseen by the European Commission General Directorate for Education and Culture. Partner institutions in the project are:

- Department of Mathematics, University of Pisa, project coordinator
- Department of Mathematics Education , University of Granada
- Department of Education , University of Lisbona.

Nowadays, in a period where multiculturalism is a characteristic element of schools, above all at primary and lower secondary level, these mathematical activities – the so-called *ethnomathematics* [D'Ambrosio] – cannot be kept out of the classroom! In view of this, and taking into consideration the results of an investigation performed by the first of these authors in the spring of 1998, the idea was developed of setting up a project which would permit material to be produced for use by lower secondary school mathematics teachers with immigrant or cultural minority pupils in their classes. The IDMAMIM project, approved by the European Commission and aimed at training lower secondary school mathematics teachers with immigrant or cultural minority pupils in their classes, was the outcome of this idea.

The partners have performed many parallel activities in the framework of this project. Since the project was principally directed at teachers, they were the first to be addressed, through the use of questionnaires [Favilli, César & Oliveras] and interviews [César, Favilli & Oliveras] aimed at defining the situation, with particular reference to the teachers' attitude and behaviour, and to needs, lacks and/or difficulties identified and noted by them.

Some of these teachers went on to collaborate in the experimentation of the didactic proposals set up as part of the project; the proposal used the *microproject* model as reference [Oliveras, Favilli & César] which is based on theories of intercultural mathematics teaching. The ethnomathematics program developed by Favilli [2000] was seen as a fundamental aid (see above).

Some comments made by teachers and pupils in Italy.

The Italian survey consisted of 107 questionnaires, 13 interviews and an experimentation performed with the assistance of five teachers.

Since the responses in the questionnaires and interviews of the mathematics teachers with immigrant pupils in their classes both highlighted:

- the lack of teacher training courses for teaching mathematics in multicultural contexts and
- the need to have some examples of intercultural teaching activities specifically for mathematics,

it was important to try with these five teachers to obtain a preliminary idea of the validity of the products created as part of this project and the microproject prepared in Italy in particular.

The Italian didactic proposal [Favilli, Oliveras & César] was based on the construction of a *zampoña* (Andean flute or Pan pipes): the five teachers were provided with an outline to use to in class; they were provided also with a very detailed but neither prescriptive nor restrictive description of the various phases (of construction) and mathematical activities (explicit and implicit) identified and indicated by the Italian project group. The teachers were therefore left with the possibility of making free didactic decisions, both when they decided to follow the given indications and when they decided to follow alternative didactic paths. From a certain point of view, it was this second aspect which was of great interest to us, in that it demonstrated how the didactic application of the project was not unique: in fact the way could (and must, for the reasons cited above) be conditioned, both in terms of professional sensitivity of the individual teacher and by the class context the teacher is working in. In addition, their is a wide range of potential for development of the zampoña microproject. In fact its application is much wider than that imagined when the craft project was conceived for use in the microproject.

Without focusing on the details of the project, [Favilli, 2004a], we intend to focus in this article on the comments made by the Italian collaborating teachers at the end of the didactic experiment. Although we cannot pretend that their comments have any quantitative value, they provide real qualitative indications of interest and positive indications in terms of perspective.

To be able to better organise the collaborating teachers' comments , they were asked to describe the various activities performed (and how they were performed) and to answer a brief questionnaire featuring ten open questions. The teachers were left free to add any other observations they considered useful.

The first question dealt with the pedagogical aim of the decision by the teacher to take part in the IDMAMIM project by inserting the zampoña activities in their teaching programs. Three teachers wanted to highlight the opportunity of permitting their pupils to work in groups collaborating together: an element all too often absent in Italian schools, especially in mathematics classes, where individual ability is excessively emphasised.

- *The main aim was to actively involve pupils in their learning process; it was fundamental to stimulate the team spirit in the pupils both amongst themselves and with the teacher.*

- *To affirm how disciplines can be transverse and the knowledge of different realities from those the pupils live in and are familiar with, to promote socialisation and equal collaboration in the group.*
- *A chance to work in a group. Multidisciplinary offer.*

As can be seen, the teachers grasped the valuable opportunity of performing a multidisciplinary teaching activity, which in effect permitted the involvement of other teachers including those of music, natural and experimental sciences, history, geography, humanities, art, etc.. In the words of one teacher, there was also a first hint of intercultural education.

It is worth noting that the same aspects grasped by the teacher were also picked up on by the students, as shown from the results of a short open answer questionnaire set to them at the end of the experimental teaching activity.

- *What I really liked about the zampoña lessons was the way we all worked together and the new experience.*
- *The thing I liked most of all about the zampoña lessons was being able to work all together: we were a real team, just like a real family; I also liked it when we found the mathematical law, because we were all enthusiastic, we felt like …important mathematicians.*
- *The zampoña lessons were interesting because it's good fun doing mathematics like this.*
- *It's a good idea to bring two such great materials as mathematics and music together in the same task.*

The teachers were also asked to indicate whether the activity responded adequately to set pedagogical aims:

- *I think so, or at least, it has certainly made a contribution, together with the other activities, to reach the aims.*
- *The children worked well in groups and collaborated well.*
- *Yes, because the group worked well, it amalgamated, there were not prevarications and manual work was re-evaluated, which for some pupils is much more fruitful than abstract thinking.*

Exactly, manual activity! The pleasure of education in and by manual activity has been lost in schools (at least in Italian schools): this teacher quite rightly implicitly implies that the school is required to offer all pupils the chance to develop their abilities, both rational and manual. At least for the duration of the period of compulsory education …

Again, the pupils' remarks confirm those observed by the teachers, showing great enthusiasm in the experience, above all in the fact that they had actually created something:

- *The thing I liked about the zampoña lessons was that … now I know how to make one!*
- *The thing I liked about the zampoña lessons was seeing something we had made working, and working well because some of our classmates even played a tune with it.*
- *The work was good fun and, to tell the truth, I really like manual work.*

Another question asked the teachers to indicate what mathematical topics they had chosen to introduce using the zampoña didactic project: most teachers indicated proportionality and functions, but other topics also included ratios, measurement tables and graphs, the basic elements of statistics, cylinders etc. One didactic aspect continuously underlined was the possibility of linking mathematics to real life problems.

The following interesting comments were made to the question asking whether there is a need to adapt a didactic proposal before being able to use it:

> *I think that any teaching proposal has to be adapted to a specific class; in fact, proposals are often presented as being suitable for an average or ideal class, this can't completely foresee the particular dynamics that develop in a class …I adapted the project as it went along to create a deeper involvement of the pupils, following their requests / needs / preferences as far as possible… for example, I had not considered introducing concepts such as averages (which had only been seen once in the science lab…), but I was guided by my pupils questions to spend time on this … this became an added objective.*

The teacher had picked up on the spirit of the didactic proposal: its ductility, the indispensable centrality of the pupil (a subject that is not simply a passive observer of his or her educative process, but actually active) and the consequent necessary ability of the teacher to manage unexpected didactic situations.

Someone noted that they had to limit the activities that could be proposed in the context of the microproject due to a lack of time:

> *No, I only had to limit it due to lack of time.*

The theme of lack of time was also present in the answers to the question that asked what difficulties were present when piloting the didactic proposal:

- *Time: inserting the proposal in the didactic program.*

- *Mainly difficulty linked to a lack of time available and how the time was made available (an hour at a time) prevented all the programmed activities from being performed.*

The time question is very interesting because it highlights how teachers (and not just those of mathematics) still very often see official programming indications (and therefore the hours required to cover them completely) as a strong constraint from which they find it difficult to remove themselves whatever their best intentions may be. Even the timetabling of the various lessons still seems for some to be an insurmountable problem.

The questionnaire couldn't be without a question asking whether the pupils had learnt significantly from the activity:

- *In cognitive terms, the activity certainly contributed to favouring pupils learning with respect to the content that had been set. Results in the test that followed were quite good.*
- *Yes , there were good results in learning about proportions.*
- *Yes, because they were very involved and interested, and were therefore capable of understanding and appropriating the topics better. Topics such as: proportions, proportionality, functions, and the concept of ratio.*

Certainly, the collaborating teachers cannot be considered to be totally objective in evaluating a project in which they played an active part!

We think we are authorised however to hope that their colleagues that will use the CD-rom containing the didactic proposals (as well as some training elements on intercultural teaching and interdisciplinary mathematical teaching) will find some ideas to reflect on that will permit them to render their educative activity even more effective for all their pupils, whatever their socio-cultural background.

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THE ELECTRONIC YUPANA: A DIDACTIC RESOURCE FROM AN ANCIENT MATHEMATICAL TOOL

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In the paper, we introduce an electronic version of the yupana, the Inka abacus. One of our main aims is to show that it is possible to make attractive and usable ancient mathematical artefacts, which still clearly prove their didactic utility. The electronic yupana, in our view, represents an attempt to link tradition and modernity, indigenous and scientific knowledge, poor and rich cultures. It aims to represent an educational environment, where pupils and students can find a friendly tool throughout which they can achieve the notion of natural number, compute basic operations, familiarize with positional notation and base change and develop personal "computational algorithms".

Introduction.

One of the main issues in mathematics education is about how to relate theory and practice. Sometimes it is quite difficult to make the right choice and find an appropriate way of dealing with both aspects. An even harder task appears to be faced by mathematics teachers who set their educational activities in the framework of the ethnomathematics programme. In fact, when planning and developing the class activities, teachers dealing with different cultures in the classroom should be aware that:

- in the past, several non-western societies greatly contributed to the development of the mathematical knowledge (see, for example, [Joseph]);
- many indigenous societies have developed and are still developing mathematical activities which, although differing from the standard ones, give those societies the necessary and sufficient tools to deal with their basic life issues (see, for example, [Bishop]);
- this mathematical knowledge is culturally relevant, thus interfering with any teaching/learning process developed in different cultural contexts,

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such as schools in *western* countries (see, for example, [Favilli and Tintori]);
- the intercultural approach appears to be the most appropriate and effective educational model [see, for example, [Oliveras, Favilli & César]) and summarizes the three previous remarks.

The intercultural approach in mathematics education is, therefore, not an easy way of teaching; it requires from teachers non-standard professional baggage and discipline knowledge, a big concern about the cultural context of the class and a great ability to adapt the curriculum to such a context. In many countries, teachers complain they can still get poor assistance both from in-service training and didactic resources specifically designed for teaching mathematics in multicultural classrooms (see, for example, [Favilli, César & Oliveras]).

On the other hand, to prepare appropriate material for intercultural mathematics education in western schools is a real challenge. In fact, it requires the correct use and full evaluation of possible contributions from different cultures to make their use both proficient, at discipline level, and helpful, at social level, to the whole class and all pupils aware that each culture has contributed and can still contribute to any kind of knowledge development, including the mathematical one.

Claudia Zaslavsky (1973), Paulus Gerdes (1999) and others have already provided us with many beautiful examples.

In the present paper, we introduce an electronic version of the *yupana*, the Inka abacus. One of our main aims is to show that it is possible to make attractive and usable ancient mathematical artefacts, which still clearly prove their didactic utility. The electronic yupana, in our view, represents an attempt to link tradition and modernity, indigenous and scientific knowledge, *poor* and *rich* cultures.

There is very little information about yupana and its use, mainly because the Spanish *conquistadores* destroyed most Inka cultural heritage. The only available representation of a Yupana is part of a design drawn by the Spanish priest Guaman Poma de Ayala (1615) in his chronicle of the Inka empire submission. In Fig. 1, the yupana is represented together with the *quipu* (a statistical tool made by knotted strings). Only recently, mainly thanks to Marcia and Robert Ascher (1980), mathematics researchers and historians have focused their attention into such mathematical instruments from the Inka culture. As far as we know, mathematics educators have paid poor attention to them so far…

Fig. 1 – ***The* yupana *(with the* quipu*), as reported by Guaman Poma de Ayala***

As it happens with all the other abaci, the yupana gives pupils, at first, the opportunity to appropriate the concepts of quantity and natural numbers, to learn their positional representation and to understand the meaning of adding and subtracting natural numbers. Other mathematical activities made also possible by the yupana (and other similar abaci) are the multiplication and the division, the representation of and the operations with decimal numbers, the representation of natural number in different bases and the changes of base.

The electronic yupana aims to represent an educational environment, where very young pupils can move their first steps into the mathematics world in an amusing and friendly way; it could be seen a very attractive, interactive, colourful, educational and easy to play game ... while learning basic arithmetic! More grown up students can find a friendly tool through which familiarize with positional notation, base change and the development of personal "computational algorithms" to perform more complex operations.

From wood to silicon – the design of a didactic computer yupana.

In Fig. 2, we show both an ancient and a modern yupana made by the street children of the NGO Qosqo Maki[1].

[1] Asociación Qosqo Maki, Calle Fierro 525, Cusco, Peru - qosqomaki@terra.com.pe.

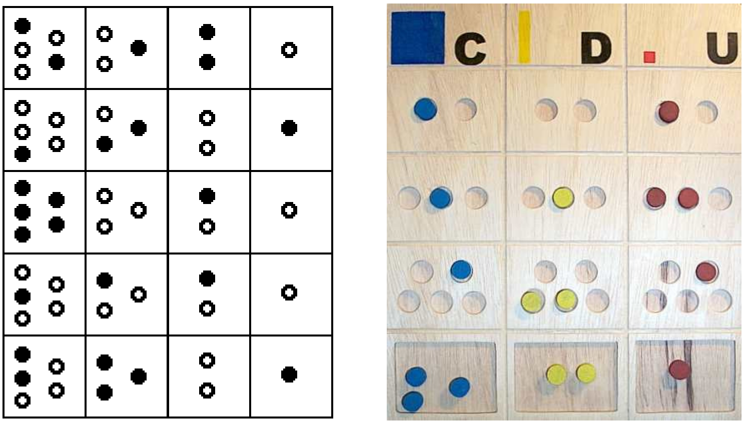

Fig. 2 – ***Ancient and modern yupana***

Goutet and Alvarez Torres (2002) have described a possible computational use of the modern yupana.
In modern yupana, numbers are represented as configurations of wood pieces on the board, using different colours for units, tens and hundreds. In the lower part of the board, rectangular areas are used either as a pieces repository or as the starting place for the second operand in arithmetic operations. As far as didactics is concerned, the presence of these rectangular areas is a weak point of the yupana. In fact, these areas allow a different representation for the same entities (the numbers and the digits) and can be confusing for children whose concept of number is developing.

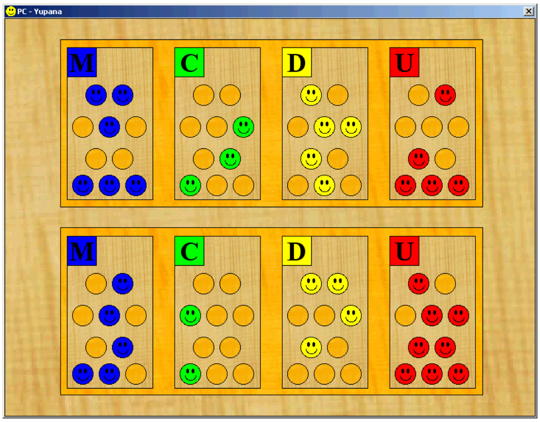

Fig. 3 – ***The double yupana***

In the implementation of an electronic yupana, we overcome such difficulty by considering a *digit* only a whole column configuration. Consequently, our computer yupana is made of *two* complete traditional yupanas, as shown in Fig. 3. In this way, all operands are represented in the same manner and both are immediately visible in the *double yupana*. For instance, the double yupana in Fig. 3 represents the numbers 6355 and 5248.

A different unifying approach arises from the only *modus operandi* that the computer yupana allows and enforces: the *drag-and-drop activity*. By dragging *one piece at a time*, the concept of number is induced by repetition of unitary increment/decrement steps.As in the wood yupana of Fig. 2, the pupil is introduced to positional notation/arithmetic with the help of colour correspondences between the two yupanas: pieces with the same positional weight are given the same colour. This allows the user to perform positional arithmetic by moving pieces of the same colour to equivalent positions. In fact, the program allows drag-and-drops involving pieces/holes with the same positional weight **only**. In this way, positional arithmetic is actually a by-product! Moreover, the teacher can change the colour scheme to avoid unconscious colour/weight associations.

The drag-and-drop activity, coupled with the general statement "*operation is over when one of the two yupanas is empty*", provides another simple unifying framework for three basic mathematical operations: sum, subtraction and base change.

The sum is performed by dragging all the pieces from one yupana to the empty spaces in the other one. Whenever a yupana is empty the pieces in the other one represent the result.

Subtraction is accomplished by "eliminating" pieces with the same weight on both yupanas, i.e. by dragging pieces to equivalent pieces. It is worth noting that, in this case, the (upper or lower) position of the yupana that is empty at the end of the process gives appropriate information about the sign of the result!

Base changing is executed with the same rules as the sum, with two main differences:

- the number of positions (the holes) for each digit on the two yupanas is different (any base between 2 and 10 can be used);
- more colours are normally involved, leading to unitary operations only in the worst case (when the two bases are co-prime) but also to interesting "diagonal drags" (as in the case of bases 2 and 4, when a piece of weight 4 is moved).

These features make the electronic yupana a solid mathematical tool upon which a child may build his/her own mathematical foundations in his/her most appropriate and distinctive way: playing.

Using the electronic yupana

We shortly report and comment three screenshots showing how to setup an operation (a sum in our case) and how to perform it with our yupana. This will clarify both the approach and the way the program reacts to pupils' actions.

In Fig. 4, we see the initial setup phase of the yupanas. This is accomplished by dragging pieces from the big smiling faces (*the sources*) on the left (in the picture a blue piece is still being dragged). When the user is satisfied, he or she may click on one of the blue "operations" on the right switching to the operation phase.

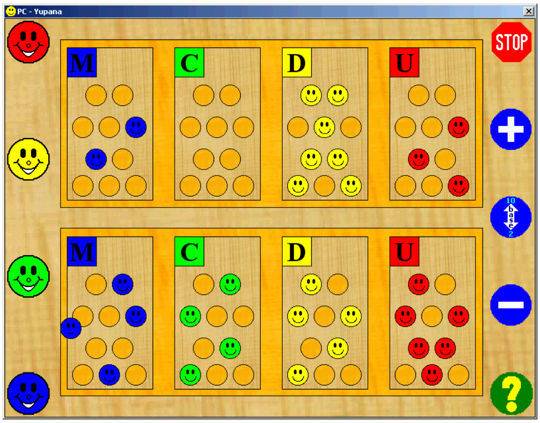

Fig. 4 – ***Setup of the board by dragging***

In Fig. 5, the plus sign has been clicked at the end of the setup phase and we are in the operation phase. The sources on the left have disappeared as like as all the other operation choices. The sum is going on and many yellow pieces have already been dragged from the upper yupana to the lower one, which is now full. A small hand has appeared signalling that a "promotion" is allowed turning all the yellow pieces into a green one (this operation is automatically performed by clicking on the hand).

All the red pieces from the upper yupana are now in the lower one, except for the last piece, which is still being dragged (…look at the hand that allows a "demotion" of a yellow piece in change of 10 red ones in the upper yupana: a similar operation is also possible with blue and green pieces).

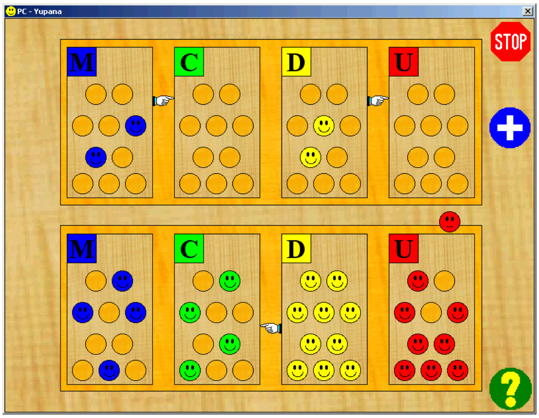

Fig. 5 – *Sum is in progress*

In Fig. 6, we see the final step of the sum. The promotion has been done allowing dragging the remaining yellow pieces to the lower yupana. All the blue pieces have also been moved (…the last one is still half way).

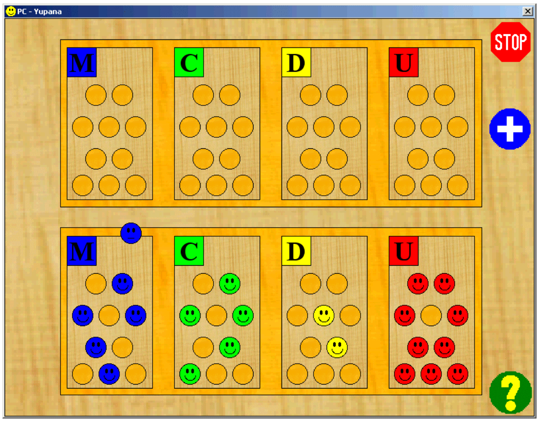

Fig. 6 – *The final step. The last blue piece is approaching its position*

Further features and aims of the electronic yupana.

In section 2, we have focused our attention on how to reinvent the yupana to obtain an easy and solidly founded didactic tool. Here we report a short list of other electronic yupana features and aims to make it an even better tool.

The program may provide classical base 10 numeric feedback by continuously showing all relevant numeric information along the border.

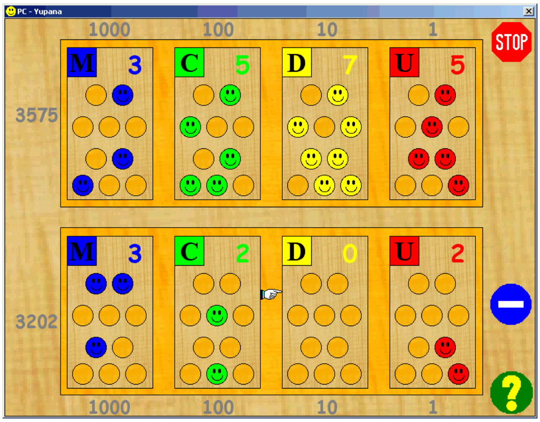

Fig. 7 – ***Numeric feedback on the yupana***

In Fig. 7, the grey numbers on the border represent the values of each yupana block (the digits), the weight of each positional digit and the value represented by each yupana as a whole (the numbers). This should provide a more friendly approach to digits and multidigits numbers.

A program option enables the “errorless learning” approach by means of visual feedbacks for the drag-and-drop activity. For instance, a piece will show a happy face when it is dragged to a position where it may be correctly dropped and a sad face in the opposite case. This allows a simplified interface for younger children and a gentle approach for disabled boys.

The electronic yupana is able to perform automatically any operation, either by unitary movements or by optimized positional moves, and may show animations showing all correct steps towards the solution.

By clicking on the question mark on the lower right of the board, the pupil may ask for a hint which will be shown by the “blinking” of the pieces or holes involved in the next correct move. The program has options to automatically suggest hints after a given period of inactivity.

The didactic exploitation of the electronic yupana is under investigation in cooperation with a few primary school teachers.

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THE ART OF TILES IN PORTUGAL AND BRAZIL: ETHNOMATHEMATICS AND TRAVELLING CULTURES

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This paper discusses some aspects of the relationship between Mathematics Education and art, focusing mainly on the study of the Portuguese tiles, which were brought to Brazil in the colonial times. In Brazil they were re-appropriated in a special way and later on came back to Portugal influenced by that hybridized form. The article shows the curricular implications that can be established through the links between pedagogical processes involving isometries and the fruition of art.

Issue for debate: Taking into account hybridized cultural processes such as the one discussed in this paper, what meanings can be given to the cultural dimensions of Mathematics Education?

Keywords: Mathematics Education and art; Ethnomathematics; Mathematics Education and culture.

1. The art of Portuguese tiles and hybridized cultures

The art of tiles is one of the cultural manifestations that throughout History have been relevant for different peoples and social groups. As any cultural artifact, it has been marked by the dimensions of conflict and struggle for the imposition of meanings. Considering tiles from this perspective implies examining the very notion of culture that supports it. Obviously, it is not a matter of thinking about it as something consolidated and fixed that is transmitted like so much "baggage" from person to person or group to group. On the contrary, as argued by authors such as Hall (2003), culture is not an archeology or simply a rediscovery, a return trip, it is production. With this meaning and following Hall, the art of tiles is understood as a cultural artifact that is concomitantly produced by and the producer of cultures, the result of a set of practices of meaning that are permanently updating and reworking themselves. It is this incessant and conflictive process of re-presentation – which is expressed in churches, convents and palaces built in previous

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centuries and also in buildings erected during the last few decades in Portugal and Brazil – that renders the art of tiles an interesting element for analysis in the field of education.

Walking through Lisbon, one finds contemporary tiles at the Metropolitan train stations built in the 80s of last century and at stations built later, showing their potential to update themselves as aesthetic support. It is precisely this innovative dimension that is found in the work produced by Ivan Chermaveff for the Lisbon Oceanarium, in which are incorporated elements of marine fauna in ensembles of hand-painted patterned tiles. It is hardly surprising that the techniques used in manufacturing the tiles have undergone change over time, related both to technological advances and to economic interests involved in their marketing. During the Portuguese Colonial Period, the art of tiles traveled from the Metropolis to colonies such as Brazil, as part of the cultural domination process whose repercussions are still felt today. This domination process, according to Silva (1999), required besides economic exploitation, also cultural affirmation, i.e., the transmission of given forms of knowledge. As the author writes: "the 'primitive' cosmic vision of the native peoples had to be converted to the European and 'civilized' vision of the world, expressed through religion, science, arts and language and appropriately adapted to the 'stage of development' of the people submitted to the colonial power"(Silva, 1999:128). However, the author stresses that cultural domination processes such as that carried out by Portugal over Brazil cannot be considered a "one-way road", i.e., the cultures of the colonial spaces are immersed in power relations in which both the dominant culture and the dominated one are deeply modified, in a cultural hybridization process. Hall (2003) also points out that the colonial logic may be understood by what Pratt calls a transcultural relationship through which "subordinated or marginal groups select and invent based on the materials that are transmitted to them by the dominant metropolitan culture" (Pratt, apud Hall, 2003:31). From this perspective, it is considered that cultural relations established between the colony and the metropolis cannot be conceived as movements of simple transmission and assimilation or else source and copy, constituting static and unilateral processes. Instead, movements that are both for appropriation and for re-appropriation of cultural artifacts in the colonial process are produced in these relationships.

One such movement – marked by transculturality, as conceptualized by Pratt – can be observed in the art of tiles which, having traveled to Brazil with the European colonizers, was re-appropriated in a peculiar manner in the then colony, when tiles began to be used mainly to cover façades, as distinct from

Portugal where, according to Calado (1998), they were used up to the eighteen hundreds basically to cover interior walls. According to Santos Simões (apud Silval, 1985:87), “the new fashion of tile-covered façades came from Brazil to the old metropolis (...) a curious phenomenon of inversion of influences”.

Thus it is observed that the use of tiles on façades in Portugal is a cultural practice that has its roots in Brazil, a practice that was transferred from the colony to the metropolis. This re-appropriation of the art of tiles by the tropical country of the then Portuguese colony, may be understood as due to one of the properties of the material that made up the tile: its use provided protection to the buildings against the erosion caused by heavy rains, reducing the indoor temperatures of houses by reflecting the sunlight. It was possibly this protection against the great in Brazil that favoured the peculiar re-appropriation of tiles, so that part of the history of the art of tiles in the Western world was constituted by two movements: the first, when it was brought from Portugal to the colony, and the second, when there was the appropriation, in the metropolis, of the peculiar way in which the tile was used in the colony, in a hybridization process that also points to the own ways in which each culture handles and deals with art and aesthetics. As says Silva (1999:129), “hybridism carries the marks of power, but also the marks of resistance”.

The influence of Portuguese tiles in Brazilian art can be observed along the coast of the country. A significant proportion of the tile ensembles in Brazil was not preserved, except for the historical center of São Luís, in the state of Maranhão, considered the “city of tiles”, which has preserved over two hundred buildings decorated in this manner.

The cultural importance of tiles in Portugal and its former colonies such as Brazil, points to a few issues related to Mathematics Education. Many studies have established these ties, using art to teach notions and concepts of Geometry. Among the authors who have discussed this issue are Martin (2003), Silva (1998), Liblik (2000) and Frankenstein (2002). Studies by Frankenstein deserve special mention, since besides emphasizing the mathematical aspects involved in the art, they problematize the cultural, political and social dimensions necessarily implied in artistic production to a greater extent. It is from this perspective that the present study was performed, as part of the Ethnomathematics perspective.

2. Ethnomathematics, school curriculum and the art of tiles

This study – in recovering elements of the history of the art of Portuguese tiles, examining the specific mode of re-appropriation of this art as performed in its colony, Brazil, and the hybridization process undergone by the tiles on returning to the metropolis – indicates three issues that are directly implicated in Mathematics Education.

The first of them concerns the cultural hybridization process. Different from the more deterministic perspectives that discuss the relationship between the metropolis and its colonies, understanding this process as a mere imposition in the sphere of the social, the economic and the cultural worlds of dominant groups over the dominated ones, this study about the art of tiles showed that this process was not limited to a mere subordination, a mere repetition of overseas culture in the colony. There is a sort of re-invention of the invention, which transform the art of tiles in Brazil into ‘another" art. But the hybridization process of the art of tiles does not end there. On its voyage back to the metropolis, it is already this "other" art that returns home. Emphasizing this discontinuity, this fragmentation, this permanent re-invention process, that does not seek "authenticity" to despise the copy, indicates a broader and more complex understanding of what "culturalizing Mathematics Education", i.e., its cultural dimension could be.

The second issue relating to the Mathematics Education presented by this investigation concerns the possibilities of establishing close connections between it and the field of History, by studying the art of tiles. At least as regards the former colony called Brazil, the complex hybridization process involved in this art has been systematically silenced in school curricula. Narratives about the colonial period have limited themselves to a political and economic vision of imperialist domination, leaving aside its cultural and aesthetic dimensions. The art of tiles may constitute one of the possibilities of incorporating these dimensions into the school curriculum, emphasizing the hues, nuances and tensions involved in the history of colonization processes. Ultimately, it is to subvert the narratives of the hegemonic colonizing discourse.

The study indicates a third issue referring to Mathematics Education. Here, it is important to examine the possibilities of seeing the plan isometries – a set of knowledges that are part of the school curriculum in the West – not "in and of themselves", not essentialized, but as mathematical tools with a potential to favour the fruition of art. Maybe one could think of art and other fields of knowledge as equally "worthy" of escaping the sidelines on which the modern school has placed them. Maybe one could think of the isometries of the plan not only as “mere”mathematical contents, but also as tools that make it possible to sharpen aesthetic sensibility, to educate the gaze. To educate, but not to domesticate it. Indeed, it is not a matter of thinking of the fruition of the art of tiles as something to be disciplined, domesticated by the composition of rotations, translations and reflections. Maybe one could think of other forms of “pedagogizing the art of tiles”, a pedagogization which would not ultimately reduce Mathematics Education to a hierarchized set of contents. Maybe one could, within the sphere of Mathematics Education, think the unthinkable, producing possibly other ways of being in the world and giving meaning to Mathematics and to art.

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ETHNOMATHEMATICS IN TAIWAN—A REVIEW

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Multiculture has been a trend in educational reform around the world. Fortuantely, Taiwan is not exempted from this trend. Indeed, multicultural curricula have been implemented in Taiwan from the elementary to college level of education. However, when compared to the concept and implementation of multicultural curricula, ethnomathematics appears to be an emerging new concept and has not been extensively studied yet. As a result, ethnomathematics in Taiwan is a field worth of our attention and investing more research endeavor if we want to increase our knowledge about the development of mathematics in different cultures around the world. The purpose of this article is to describe the development of ethnomathematics in Taiwan. It contains four parts: 1)a brief introduction of Taiwan; 2)research on Taiwanese indigenous students; 3) math studies of Taiwanese indigenous students; and 4) suggestions for the development of studies in ethnomathematics in Taiwan.

About Taiwan

Politically, Taiwan is a democratic country of which people have practiced the elections since 1972 and direct presidential election since 1996, albeit the fact that Taiwan underwent the longest history of martial law in the world (1949-1987, 38 years, under the Chiang family's regime) (1)). Taiwan is an island located in South-East Asia, 62.5 Km (100 miles) off China's coast between Japan and the Philippines. Although it is very close to China, Taiwan is <u>not</u> part nor one province of China. Taiwan has an area of approximately

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36,000 square kilometers (13,800 square miles), which is approximately the size of West Virginia of the U.S., or somewhat smaller than Netherlands. Taiwan consists of less than 1/3 of plains, whereas hills and mountains take up the rest of the area, in which 23 millions of people reside (70% Holo; 14% Hakka; 14% "mainlander; and 2% Indigenes) and about 58% of population live in urban areas(2). Taiwan is ranked as the 45th populated country among 192 countries in the world (3).

In the past 400 years of history in Taiwan, Taiwan has been successively controlled by foreign rulers. The colonial rulers included the Dutch (1622-1661), Spanish (1626-1642), Koxinga (1661-1683), Ching (1683-1895), Japanese (1895-1945), and Koumintang (KMT) (1945-2000) regimes (4). As the result, the indigenous cultures and languages were repetitively suppressed by the rulers.

The name of 'Taiwan' originated from the Siraya tribe, one of the Pingpu (Plain Indigenes) groups which is extinct as 'Taian' or 'Tayan,' meaning 'outsiders or foreigners' [5]. It was the Portuguese (1557) that named Taiwan as 'Ilha Formosa,' meaning 'beautiful island' [1]. In the 17th century, the Dutch called Taiwan as 'Taioan' [1, 5]. The names for Taiwan have evolved with different immigrants or invaders so that Taiwan was given different names along the history. Yet, the differences were basically due to the variations in the pronunciations and translations of different languages. The fairest guess shall be the name used by the Pingpu when they saw the outsiders or foreigners and cried out to their tribal people ''Taian' or 'Tayan.' Gradually, with the time the "foreigners" of Taiwan thought it is the name for this beautiful island.

The names for the Indigenes in Taiwan changed at different times of colonization. However, the names given by the 'outsiders' all shared the same connotation that the Indigenes were lower class, uncivilized, uncultured and/or less educated than the new settlers.

Dating back in the 17th and 18th century, the Chinese immigrants called the Indigenes in Taiwan as ‘fan1’ (番), meaning barbarian. It was an insulting word with the ego-centric view that the Han (Chinese) culture was the best. (China in Mandarin Chinese means ‘the center of the world’). With the increasing encounters with the Taiwanese Indigenes, the Chinese started to realize that the Indigenes consisted of more than one group or one kind. Therefore, the Indigenes were classified based on the living areas. Those living in the mountains or in the East (of which area was not developed in the beginning of colonization) were named ‘high mountain fan’ whereas those on the plains were named the ‘Plain fan’ or ‘Pingpu fan.’ Such a classification had been reserved until today for the general categorization for the Indigenes in Taiwan—‘high mountain group’ (gao1 san1 zu2 高山族) and ‘Pingpu group’ (pin2 pu3 zu2 平埔族). The Japanese adopted the classification system used by the Ching government and based on the degree of assimilation further classified the fan into ‘raw fan’ (shen 1 fan) and ‘ripe fan’ (shou2 fan). The latter means the group which was assimilated to the ruling culture [1].

Research on Taiwanese indigenous students

The Indigenes in Taiwan belong to the Austronesian. The languages used for the Taiwanese Indigenes (TI) belong to the Austronesian or Malayo-Polynesian language family, which consist of approximately 500 languages in total [1]. Such a language family comprises the highest number of languages in the world, involving approximately 170 millions of speakers. Geographically, it is also the most wide spread for this language family. The Taiwanese Indigenes are located in the most north point along this language and cultural line. According to the Australian scholar Peter Bellwood [1], Taiwan might be the starting point for the first stage of immigration of Austronesian people around the world 6000 years ago. Thus, research on TI is important and of value.

The term for the Indigenes as ‘yuan2 zhu4 min2’ (原住民) was first coined in 1995 in Taiwan after the United Nations made 1993 the year of the Indigenes [6]. It means the originally-living-here-people. The shift of naming for TI from barbarian, mountain people into Indigenes reflects the rise of human-rights and respect for multicultures in Taiwan. In 1996, the constitution had been amended and added the rule No. 10, indicating that the status and political rights of the Indigenes should be protected, and the government should support and foster the development of their cultures and education [6]. The law of the indigenous education was announced and enforced on June 17 of 1998, and Sept. 1 of 1999 by the presidential command, respectively [7].

The author reviewed the studies on Taiwanese indigenous students based on the results gained from three mostly used data systems in Taiwan [8, 9, 10]. The results showed mixed findings (see Appendix). That means, although the same key words were used, the results differed from different systems or by using different word combinations. For example, ‘yuan-shu-min math’ had less results than ‘yuan-shu-min and math’ as keywords. This suggested that the combined words might limit the research results when they did not match the stored keywords. On the other hand, the other way might inflate the results. In addition, when English keywords were used for search, ‘aboriginal’ or ‘aborigines’ showed more results than ‘indigenous’ or ‘indigenes’ did. Another finding was that the tribal names were used as keywords. In this case, such studies would not be found unless the specific keywords were used. Therefore, how to regulate the keywords, especially in English, for the Indigenous studies in Taiwan seems necessary and urgent.

As a whole, the literature on the Taiwanese indigenous students has shown the efforts to analyze the learning problems that the indigenous students encountered in school. The factors under study included family factors (e.g., economy, parents’ education), curriculum, learning styles, cultural differences in learning, and cognitive behavior.

A line of research applied different IQ or cognitive tests from the mainstream cultures to the indigenous students (IS). As expected, the IS did poorer than the non-indigenous students (NIS) on these ‘mainstream’ tests. However, such findings were depressing and not constructive because they did not help us to understand nor help solve the difficulties or learning problems that IS have in school. Such culturally unfair tests were similar to those that had judged IS to be poor learners in school in the first place.

Math studies of Taiwanese indigenous students

From the Dissertation and Thesis Abstract System [8], using the keyword of ‘ethnomathematics’ in English resulted in 13 studies, of which 6 were really related to the topic, and the same keyword in Chinese found 7 studies, of which only 4 were related to the keyword. The search of the keyword ‘yuan-shu-min math’ pulled out 15 studies, of which 12 were related to the Indigenes. As a result, there were only 16 studies found from this system that were related to ethnomathematics in Taiwan after the repetitive results were deducted.

On the other hand, the results were not optimal from the other two data bases [9, 10] using the same keyword to search. For instance, only 2 and 1 results were found for ‘ethnomathematics’ when the keyword in Chinese was typed; none and 1 study was found for the keyword ‘aboriginal math’ in Chinese. The results could be attributed to the fact that these two data bases only contained the published journal articles and the listings of the systems were not very inclusive. Since most theses or dissertations were not published so that these two data bases would pull out very few results. This suggests that more efforts shall be invested in publishing the works that have been completed relating to ethnomathematics and in conducting more of this type of research in Taiwan.

In the following section, the research up to date on the Taiwanese Indigenes in math conducted in Taiwan will be summarized and briefly discussed. The topics of the studies can be classified into 5 categories, which include number concept, arithmetic skills, problem solving, geometry, algebra, and cultural differences.

Generally speaking, math is considered the most difficult and unfavorable topic for the indigenous students (IS) in Taiwan. Yang [11] studied the 5^{th} graders in Taitung area (East Taiwan) and found that the IS fell behind their non-indigenous peers (NIS) about 15 points on average, on a full scale of 100 points, in math. Many studies indicated that the gap in math achievement between the IS and the NIS increased along the school years. The studies from the search results for how and why this gap exists in different areas of math will be described in the following paragraphs.

Number Concept & Arithmetic Skills

There are 3 theses and 1 journal article found on the topics of number concept and arithmetic skills. Tsai [12] studied 12 senior level elementary students of Makatau (6 for the 5^{th} and 6^{th} grades, respectively) on the relationship between the math attitude and achievement in decimal and geometry concepts. Makatau is a division of the Pingpu Indigenes that are located in the south-western of Taiwan. Tsai found that the indigenous students (IS) scored significantly lower than the non-indigenous students (NIS) on all the test measures, including the attitude scale. Tsai concluded that the math achievement correlated to the math attitude of the students.

There were 2 studies on the number concept of Atayal children [13, 14]. These two theses were from the same advisor so that the design was almost identical, except for the number and grade level of the subjects. Therefore, the findings are summarized together. The first grade Atayal children had difficulty in the concept of zero, ordinal numbers, comparison problems, and mental arithmetic. Their number concept and counting skills were less mature than their non-indigenous peers. In general, the Atayal children lacked the flexibility to represent the combinations of the numbers (e.g., 10 can be made up with different combinations)[13]. When the Atayal children in the 2^{nd} grade, they showed improvement in number concepts, counting skills, number representation skills, number reservation, and the concept of zero. However, when compared to the NIS, the Atayal children still fell behind on categorization, the concept of zero, ordinal numbers, and number representations [14].

The conclusions drawn from these two studies were that the differences in the number concept achievement for the Atayal children were attributed to the cultural differences. The number concepts of the Atayal first graders primarily relied on the family education than the preschool education. Sharing and especially equal sharing were the norm in the Atayal culture. Although the Atayal number system was a base-10 system, large numbers and ordinal numbers were seldom used in daily life. So it did not help much in the Atayal children's counting ability. In addition, the concept 'zero' did not exist in the Atayal language. As the result, the number concepts that the Atayal children demonstrated corresponded the value and experiences they had from their culture and daily life.

The conclusions from the two authors sounded more as a tone of 'shortage' or 'lacking' than how to respect the differences as a starting point and then how to use the differences wisely to educate both sides of the students—the indigenous and non-indigenous.

Lin [15] studied a total of 135 students from three grade levels (2^{nd}, 3^{rd}, and 4^{th} grades) in three groups on the place value (Indigenes in the mountain area, Non-indigenes in the mountain, and Non-indigenes in the city). The 2^{nd} and 3^{rd} grade indigenes scored significantly poorer in the oral counting and object counting than the two groups of non-indigenes. There was no difference between the two non- indigenous groups. So the author concluded that it was not the 'area' (in mountains) caused the achievement gap, but the ethnic group. I think such a conclusion was not solid and also biased against the indigenes because it lacked the 4^{th} comparison group, the Indigenes in the city for the study. An inference like this was at risk of making a prejudiced comment on the indigenes that they were ‘not smart as the non-indigenes.’ In addition, the finding that the difference in the place value achievement disappeared for all the 4th grade groups seemed deserving further explanation from the author. In fact, no information on the ethnic groups made the findings less conclusive and valuable.

Chien [16] observed the Yamei (now called Dao) villages in the Orchid island (Lanyu) for the daily activities. The Orchid island was located in the pacific and very close to the Philippines islands. Chien concluded that the cultural differences were the contributing factors for the learning difficulties in math and low motivation in learning math for the Dao children. For example, the number system in Dao language was complicated and without written symbols. There was no real economy in the island. The traditional life was mostly self-sufficient that men’s job was fishing and women’s job was farming on the hills. Therefore, no money system in the past and even today the opportunity was still scarce for the children to use money and big numbers in daily life. In addition, sharing instead of competition, emphasized in school, was valued in the culture. As the result, school education was not valued by the parents on this island. In summary, Chien found that Dao’s cultural values which were different from the mainstream or school culture have caused the learning problems in math. However, I find that Dao’s number system was not without rule, although the number words were much longer than in Chinese. Also, it is worth of further investigation whether the Dao cannot or do not want to count.

Problem Solving

This is the only thesis studying the aboriginal math underachievers [17]. The methodology adopted in this study was not different from that is commonly used for screening the ‘learning-disabled’ students in school for the non-aboriginal students. Different standardized cognitive and achievement tests were used to obtain the experimental and control groups in which 40 aboriginal students were included, respectively. No information for the ethnic group of the subjects was provided. As expected, the students in the experimental group performed worse than the control in all the mathematical measures—addition and subtraction arithmetic, and word problems. The errors made by the experimental group were very similar to those of the math underachievers. However, no further explanation was given by the author. This study did not include the non-aboriginal students for comparison. Thus, it is no way to know if the aboriginal math underachievers would differ from the non-aboriginal math underachievers.

Geometry

There were 7 theses studying the concept of geometry of the indigenous students [12, 18-23]. The subjects of this group of research were mainly from two geographical areas, the north-east and south-west of Taiwan (i.e. Hualien and Pingtung county). The tribal groups included the Atayal, Paiwan, and Pingpu. One study did not mention about the name of the indigenous group. Except for one study was on the lower grade level of the elementary school [22], the rest of the studies targeted at the students at least from the 4^{th} grade on. All studies adopted the van Hiele levels to interpret the understanding and achievement of the geometry concept for the students. Two studies did not include the control group for comparison [18, 23]. Only one study indicated Holo and Hakka as the control groups [20], while the rest of the studies generally mentioned that the control groups were non-aboriginal students.

As expected, the achievement of the geometry of the indigenous students was not very high, and generally speaking was lower than the control groups. However, the information was lacking in all studies about the expected level for various geometry concepts for different age groups. As the result, all studies were full of separated findings listed, but failed to provide the reader a more meaningful comparison and interpretation about the topics they intended to study—the indigenous students' geometry concept.

Algebra

There were 2 theses studying the algebra achievement of the indigenous students [24, 25]. The subjects were the junior high school students. No control group was included in both studies for comparison. One study was about the Atayal students, while the other did not mention about the tribal name. The indigenous students had significant difficulties on algebra because they had difficulty in using the abstract symbols to represent and solve the problems.

Cultural Differences

There were 6 studies relating to the topics of cultural differences of the Indigenes [26-31]. Lou [26] interviewed 5 Atayal boys about their world view and found that the Atayal did not pay attention to the precise time, but the events. The clock time has not fully replaced the natural time; the time concept was a relative point relating to an event rather than at an exact point. Encouragement in face was not accepted in Atayal culture. The rest of the studied were the endeavors in inspecting the cultural and social contexts as factors for attributing to the different learning experiences as well as preferences for the indigenous students in the math class. Therefore, suggestions for different teaching methodology were proposed in the studies.

Suggestions for the Trend of Ethnomathematics in Taiwan

In summary, the term to replace the 'mountain people' for the Indigenes did not happen too long ago in Taiwan. The law and emphasis on the indigenous policy did not start until 1999. The research for the indigenous studies in this paper has not yielded too many results. For half of the studies, the research construct was based on the mainstream theory, logic, and methodology, although it was claimed to be the ‘indigenous studies’ because the subjects were the indigenous students. Such findings were not constructive because they did not further our understanding of the difficulties of the indigenous students, but at risk of reinforcing our biased impression that the math abilities of the indigenous students are poor. On the other hand, some studies (although very few) indicated the cultural and social contexts as the important factors attributing to the different outcomes in mathematical learning and achievement for the indigenous students. Of these studies only one thesis used ‘ethnomathematics’ and one dissertation used ‘ethnography’ as the keywords. However, up to date this small pool of studies has remained in the stage of theoretical discussions. No experiments or studies have been seen on methodology or curriculum designs corresponding to the cultural differences for the indigenous students. This should be the next move for the endeavors put into the ethnomathematics research in Taiwan.

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Appendix

Table 1: Search Results on Taiwanese Indigenous Studies with Different Keywords from the Dissertation and Thesis Abstract System

In Chinese	In English	In English
yuan-shu-min (原住民) 2418	aborigines 313	indigenes 10
	aboriginal 549	indigenous 593
	aboriginal school 11	indigenous school l0
	aboriginal people 43	indigenous people 41
	aboriginal education 30	indigenous education 15
	aboriginal student 12	indigenous student 11
	aboriginal students 63	indigenous students 6
yuan-shu-min math (原住民數學)15 (12)	aboriginal math 0	indigenous math 0
	aboriginal students + math 5	indigenous students + math 2
	aboriginal students + math edu 0	indigenous students + math edu 0
	aboriginal education + math 1	indigenous education + math 0
	aboriginal education + math	Indigenous edu +

	edu 0	math edu 0
yuan-shu-min math edu (原住民數學教育) 0	aboriginal math education 0	indigenous math education 0
yuan-shu-min + math (原住民+數學)262	aborigines + math 6 (2)	indigenes + math 0
yuan-shu-min low achievement (原住民低成就) 0		
yuan-shu-min + low achievement (原住民+低成就) 74	aborigines + low achievement 2	indigenes + low achievement 0
yuan-shu-min + low achievement/achievers (原住民+數學低成就) 10 (3)	aborigines + math low achievement 0	indigenes + math low achievement 0
	aborigines + math low achievers 0	indigenes + math low achievers 0
yuan-shu-min + low achievement + math (原住民+低成就+數學) 45	aborigines + low achievement + math 2	indigenes + low achievement + math 0
yuan-shu-min edu+ low achievement + math edu (原住民教育+低成就+數學教育) 2	aboriginal edu+ low achievement + math edu 0	indigenous edu+ low achievement + math edu 0
yuan-shu-min math+ low achievement + (原住民數學+低成就) 4 (2)		
民族數學 7	ethnomathematics 13 (6)	Cultural differences

		32
多元文化數學 4	Multicultural math 0	Cultural differences + math 5 (1)
	natives 2622	
	natives 96 (17)	
	natives + math 0	
	native + math 34	
	native elementary school 1	

*The number in the parenthesis is the actual number related to the keyword.

STUDENTS' MATHEMATICS PERFORMANCE IN AUTHENTIC PROBLEMS

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Although ethnomathematics may be different from the formal way of learning mathematics in schools, ethnomathematics should be used to further develop the contents of our school mathematics. This idea has been employed in PISA mathematics framework, which encourages linking mathematics and the real world through authentic problems. A preliminary research has been conducted to test the students' performance on different versions (authentic and non-authentic) of problems with the same underlining mathematics. The t test analysis shows that the students significantly outscored on authentic problems. This result suggests that authentic problems would lead to a better learning environment in mathematics.

Issues for debate: What exactly is an authentic problem? Should we employ authentic problems in teaching mathematics?

Keywords: Authentic problem, PISA, ethnomathematics

Introduction

One of the basic believes of ethnomathematics theory is that students can learn mathematics through their daily lives/activities. This kind of approach of learning mathematics may be very different from the formal way of learning mathematics that students encounter in their schools. However, we should not deliberately separate ethnomathmatics and school mathematics. Ethnomathematics should be used to further develop and enrich the contents of our school mathematics. This idea has been employed in the PISA mathematics framework (OECD, 2003). The formal definition of mathematics literacy of PISA is the following:

> *Mathematics literacy is an individual's capacity to identify and understand the role that mathematics plays in the world to make well-founded judgments and to use and engage with mathematics in ways that*

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meet the needs of that individual's life as a constructive, concerned and reflective citizen.

The discussion document of ICMI Study 14 (ICMI, 2003), *Application and Modeling in Mathematics Education,* describe the framework of PISA:

> *Results of the first PISA cycle(from 2000), an intense discussion has started, in several countries, about aims and design of mathematics instruction in schools, and especially about the role of mathematical modeling, application of mathematics and relations to the real world.*

The framework of PISA encourages linking mathematics and the real world through authentic questions. (Wu, 2003)

Designing Authentic Problems

Students come from the society and will go back to it. The ethnomathematics approach poses a challenge for Hong Kong teachers to establish the authenticity of the problem context because mathematics that is taught in school should be integrated with ethnomathematics. In textbooks, it is common to see "naked" drill questions, such as "simplify a fraction" or "solve a linear equation". Here are some examples:

Given $\frac{25}{7} = \frac{x}{12}$ solve x.

(1) A number is 32 larger than another smaller number. If we add 7 to both numbers, the larger one is 3 times as the smaller one. What is the larger number?

In fact we can pose another types of questions that we call them authentic questions:

(1') Three students are planning for a BBQ and going to buy some food. They know the price for 7 pieces of chicken wings is $25. Now they want to know the price for 12 pieces of chicken wings.

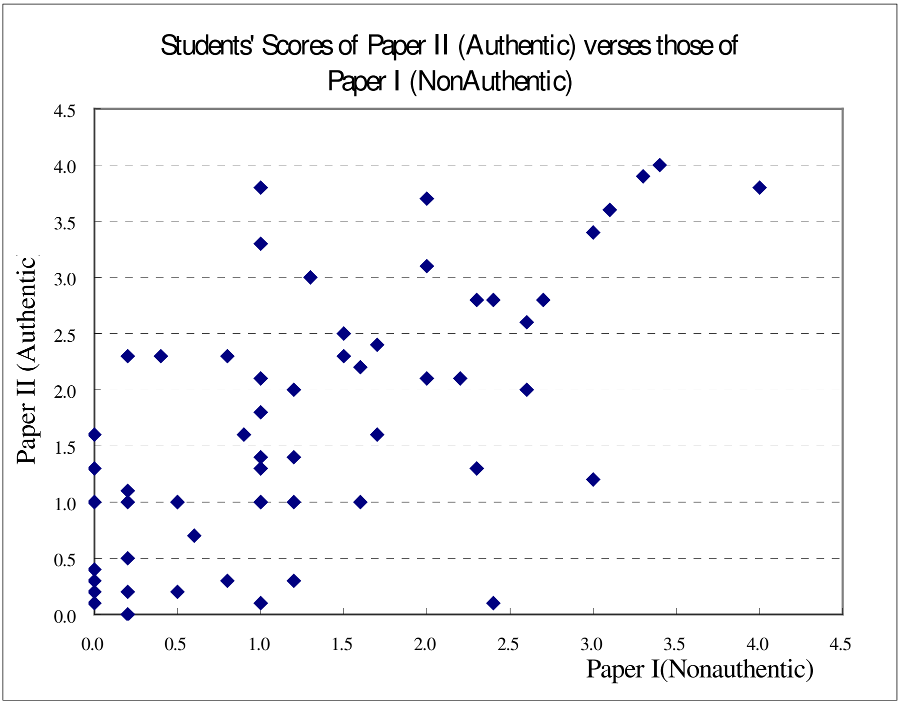
(2') A father's age is 32 years older than his son's. 7 years later, his age will be 3 times as old as his son's. What is the father's age now?

The contexts of the above two examples are different, but the underlining mathematics is the same. The second version about organizing a BBQ has a more authentic context. Would students in our school perform better in solving authentic problems? Should we employ more authentic problem in our mathematic teaching? A preliminary research has been conducted in our school to test the performance of students in different versions of problems with the same mathematics. Each student was asked to finish two papers, Paper I (Non-authentic) and Paper II (Authentic) with parallel versions of four problems. The underlining mathematics for the four problems was the same. Each student was asked to finish two papers, Paper I (Non-authentic) and Paper II (Authentic). They had four problems in parallel.

Mathematics Performance of the tests

Sixty randomly selected Secondary one (Grade 7) students were involved. Each problem carried one mark. The full mark of each test is 4. The average score of Paper I (Non-authentic) and Paper II (Authentic) is 1.27 and 1.69 respectively

A scatter plot of scores of Paper II against Paper I is shown below:



This scatter plot shows the result that students in general score more points in Paper II (authentic) than in Paper I (non authentic). The result of *t*-test analysis supports the argument on that, $t(59) = 3.53$, $p = 0.001$.

Test items performance on Authentic and Non-authentic problems

Authentic problem		Non-authentic Problem		
Mean	**SD**	**Mean**	**SD**	**t-value**
1.69	1.12	1.27	1.04	3.53**

t-value **p = 0.001**

This preliminary result shows that the average score of our students is relatively poor. The average score is below 2. However, it does indicate that students perform better in working on authentic problems. From the above plot, we can also see that data is scattered in the upper region. In other words, student did better in answering authentic questions.

Conclusion

Ethnomathematics tries to bring daily life to school mathematics, as well as to bring school mathematics to daily life (Nunes, 1992). A preliminary result in our school shows that authentic problems would link up these two things and lead to a better learning environment in mathematics. Situating the mathematical content and the mathematical relationship differently dose make a difference to the outcome of the assessments that are used to make judgments of students' mathematical competence.

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Appendix

The two Papers of problems

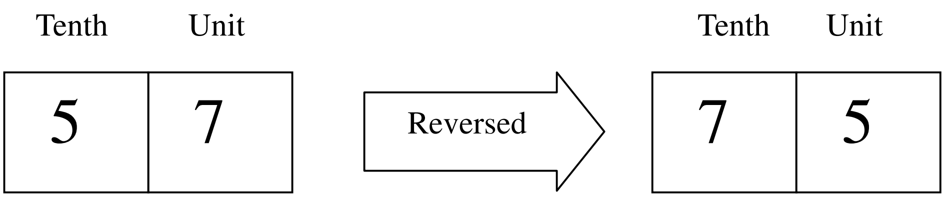
Paper I

1. A number is 32 larger than another smaller number. If we add 7 to both numbers, the larger one is 3 times as the smaller one. What is the larger number?

2. Some pencils are being packed in three different colours of boxes, namely red, green and blue. Red box can be put x pencils while each of the green and blue boxes can be put half as many as that of red box. Now there are R red boxes, G green boxes and B blue boxes. Express the total number of pencils T in terms of x, R, G and B.

3. The sum of two numbers is 100. The difference between the smaller number and 3 times of the larger number is 184. What is the small number?

4. A two-digit number and the number, which is made up by its reversed digits, have the sum 143. If the unit digit is 3 larger than the tenth digit, what is the original number? *See the example, for the number 57, the tenth digit is "5" and the unit digit is "7". The unit digit "7" is 2 more than the tenth digit "5". The value of 57 is calculated as* $5 \times 10 + 7 = 57$. *The number with reversed digits is "75".*



Paper II

1. A father's age is 32 years older than his son's. 7 years later, his age will be 3 times as old as his son's. What is the father's age now?

2. Tom and John share some \$10 and \$1 coins of total sum of \$143. The number of \$10 coins Tom has got is 3 less than the number of \$1 coins. Also, Tom's number of \$10 coins is equal to John's number of \$1 coins, and vice versa. How much does Tom get?

   Let the number of \$1 coins Tom has got be x…

3. The full entrance fee of Ocean Park is \$x dollars for adults and half price for elders and children. A group of tourists, with N adults, n elders and m children are visiting Ocean Park. Express the total entrance fee F for such group of visitors in terms of x, N, n and m.

4. A student is doing a test of 100 questions of *True* and *False* in which the score is calculated in the following ways:

   A correct answer will be given 3 marks
   A wrong answer will be deducted 1 mark.

   It is known that the student has answered all questions. And he finally scores 184 marks. How many wrong answers does he make?

   Let the number of wrong answers be x…

SOME EVIDENCE FOR ETHNOMATHEMATICS: QUANTITATIVE AND QUALITATIVE DATA FROM ALASKA

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> *When you look at someone else's culture you get more ideas—he helped us to open our eyes to see that our culture had more math in it. I am looking that way with open eyes, how they [the elders] used math to survive. (Nancy Sharp, 2003 Summer Institute, Fairbanks, Alaska)*

Nancy Sharp, an experienced Yup'ik teacher, colleague, and friend, refers to our more than two-decades-long collaboration that prizes Yup'ik Eskimo elders' knowledge through the slow and deliberate development of a culturally-based math curriculum for elementary students. From the late 1980s to the present, we have been engaged in the difficult task of connecting Yup'ik elders' knowledge to elementary school teachers and students by translating, transforming, and adapting this knowledge into supplemental math (ethnomath) curriculum.

The advent of modern schooling remains within the memory of some elders—some having been taught within the traditions of their culture and not in the Western system of education. The arrival of Western schooling coincided with colonialism and the imposition of one-way of life, one-way of schooling, and one-way of communicating (in English) to the exclusion of all things Yup'ik. More than a century's worth of literature on schooling in American Indian/Alaska Native (AI/AN) communities clearly and loudly states that the culture of the community, the knowledge of the elders, and the power to affect schooling should, at least in part, include the local AI/AN community (summarized by Deyhle & Swisher, 1997; Lipka, 2002). Grudgingly, the colonial model of schooling is slowly being transformed in Alaska and elsewhere by changes in power relations between mainstream society and indigenous cultures; one way this occurs is by acknowledging the

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importance of local cultures and knowledge, and how they can play a vital role in schooling (see http://www.ankn.uaf.edu/sop/SOPv5i5.html and the work of Ray Barnhardt and Oscar Kawagley, in particular).

Demmert's and Towner's (2003) exhaustive review of culturally based education (CBE)[1] found more than 10,000 related studies; however, out of those only a handful met the researchers' criteria of rigorous educational research. Further, although others have developed culturally-based math curriculum, sometimes derived from the ethnomathematics of indigenous groups (Pinxten, 1987; Harris, 1989; Denny, 1986), few math curricular projects have been implemented and even fewer have been researched to assess their effectiveness (Brenner, 1998; Lipka & Adams, 2004 http://acclaim.coe.ohiou.edu/rc/rc_sub/pub/3_wp/LipkaAdams20.pdf).

A Slow Transformative Process

Yup'ik Eskimo elders have their own ways of perceiving, counting, measuring, and locating in space (Jacobson, 1984; Lipka, Mohatt, & the Ciulistet, 1998). Some elders use the stars to find their way while navigating across the vast (undifferentiated to outsiders) tundra during the winter nights, and have formed a particularly rich lexicon of spatial words based on these experiences. The Yup'ik way of quantifying includes a base 20 system with sub-base five (Lipka, 1994). These brief examples of embedded mathematics illustrate a sample of the rich cultural and linguistic heritage.

Elders, in collaboration with Yup'ik teachers, educational researchers, and math educators, have come together to form an odd but effective working group with the aim of developing supplemental math curriculum for elementary school students. It is ethnomathematical in the sense that D'Ambrosio (1997) discusses how culturally different groups (from mainstream Western society) have their own ways of knowing and hence quantifying, perceiving, measuring, proving/justifying, and ordering space. The project's goals, however, are both ethnomathematical and mainstream math education:

1. to improve the academic (math) performance of Yup'ik and other Alaskan

[1] Culturally based education is somewhat of a misnomer because all education is culturally based; education composed of texts, rules, ways of communicating, valuing, and evaluating are always based on a particular culture. However, when that culture is the mainstream or powerful culture schooling is not referred to as culturally-based. It is just schooling.

students; and

2. to alter the politics of exclusion by including elders' knowledge in the school math's curriculum, thus strengthening connections between school and community.

This paper is one of the first to document the effects of a culturally-based math curriculum on indigenous students' academic performance. Systematic research can make a strong case for the inclusion of ethnomathematics curriculum. This case is made by providing evidence (quantitative data derived from a quasi-experimental research design) that Alaska Native students who engage in this culturally-based curriculum, *Math in a Cultural Context*, outperform comparably matched groups of students who use their school's mainstream curriculum. Further, qualitative data shows some of the teacher-student, student-student, and school-community effects of this curricular effort. This paper presents a brief description of the curricular construction process and its design features, research methodology, quantitative and qualitative findings, and concluding remarks.

The Curriculum Design

Developing curriculum, *Adapting Yup'ik Elders' Knowledge*[2], to meet the dual goals of improving students' mathematical performance and strengthening their cultural knowledge required an approach in which we needed to consider many factors. We did not aim to develop curriculum that could only be taught by Yup'ik people so that they could transmit culture from one generation to the next, although this is possible in some circumstances. Instead, our starting point was to analyze and examine subsistence-related "activity" such as star navigation, story knifing, kayak building, etc., and then to try to understand the underlying cognitive and pedagogical processes and connect those to schools' math curricula. For example, in many Yup'ik subsistence activities, spatial awareness, spatial abilities, and mental manipulation of objects are important cognitive components. Further, another approach we used was to examine the Yup'ik numeration system. Because it is base 20 and sub base 5, it made for a good contrast with the more typical base 10 taught in schools. In this example, we designed a module—*Going to Egg Island: Adventures in Grouping and Place Values* (Lipka, 2003)—directly from Yup'ik knowledge. Similarly, we

[2] *Adapting Yup'ik Elders' Knowledge: Pre-K-to-6 Math and Instructional Materials Development* was funded by the National Science Foundation under award #9618099. The project was funded in 1996 for four years.

incorporated Yup'ik ways of measuring, which include standard body measures applied individually to a specific purpose/task.

Elders almost always came to meetings with stories. These were highly prized—elders even made sure that they told their story before they had to go to the hospital for treatment! From this cultural activity and others, we derived the math content, pedagogical processes, and spatial and cognitive abilities. Further, students learn tasks during subsistence activities according to what they are capable of doing at the time. They work on whole processes and put things together as they understand them. Learning and the responsibility of learning is strongly placed on the student (within a cultural perspective) (Lipka & Yanez, 1998). Expert-apprentice modeling is one pedagogical form used by elders and more traditionally-oriented Yup'ik teachers. We adapted this process to the curriculum and the classroom to create a social milieu in which the more advanced novices appropriate the learning and in turn teach others (Lipka & Yanez, 1998), much like structures described by Vygotsky (see Rogoff, 1990). This process could be called a "third-way" —not necessarily Yup'ik nor reform-oriented but open to ways of accommodating math instruction in schools and communities where communicative norms differ from mainstream and Yup'ik elders.

The curriculum design approach was also influenced by the NCTM (2000) math reform agenda. Further, the curriculum includes a variety of cognitive process (analytic, creative, and practical) (Sternberg et. al., 2001.); it was organized to involve students with different abilities in a variety of tasks so that all students have an opportunity to participate and, on occasion, to lead. The hands-on design was typically connected to local activity.

Lastly, attention was paid to designing activities that invoke mathematical interest leading toward mathematical understanding. For example, in the *Fish Rack* module (perimeter and area), students are challenged to explore constant perimeter and changing areas of rectangles. Students have difficulty perceiving and understanding that area changes when perimeter is held constant, and this activity evokes heated disagreements. However, these can develop into ideal situations for students to "prove" their conjecture through mathematical modeling; the curricular design encourages the students to communicate mathematically. Yet, mathematical argument contradicts some Yup'ik values. We are still working with Yup'ik teachers on ways to establish classroom math communication that fits local values and communicative norms.

Brief Description of the Research Methodology

To answer our major research question—"Does a culturally-based elementary math curriculum (treatment) increase Alaska Native students' math performance?" —we used mixed methodologies, both quantitative and qualitative. Each methodology responds to different but highly-related aspects of the project and our overriding research questions.

<u>Quantitative Research: Quasi-experimental Approach</u>

The quantitative design is a basic quasi-experimental design (strong when we can randomly assign teachers and moderate when we cannot) in a 2 x 2 format, treatment and control and urban and rural (mostly Yup'ik and other Alaska Native students). Through pre- and post-test differences, we ascertained the effects of the treatment (the culturally-based modules) on students' math performance. From the inception of the project[3] on November 1, 2001, to December 1, 2003, we have used three different culturally-based math modules (*Building a Fish Rack*, a 6th grade supplemental curriculum that covers shape, perimeter, and area; *Going to Egg Island*, a 2nd grade math module about place values and number sense; and *Berry Picking*, a 2nd grade curriculum on measuring, data, and graphing). We have had six different trials with these curricula in both rural and urban Alaska. The results can be found below in the section entitled "Findings."

<u>Qualitative Research: Case studies, video analysis, and discourse analysis</u>

Because of the interplay between our research methodologies (quasi-experimental design and qualitative data), we used results from the quasi-experimental design to surface anomalies (unexpected cases). For example, the quantitative analysis of gain scores (student outcome measure) pinpointed certain classes, particularly those from low performing (based on previous standardized test results) rural school districts and classes that made "unexpected" gains. The qualitative component is then used to explore the classroom interaction and analyze videotape of student-student and teacher-student math communication in classes that performed well. Further, interviews with elders, teachers, and students provided other insights into the effects of this curriculum.

[3] Project refers to U.S. Department of Education, Office of Educational Research and Improvement (OERI), Award #R306N010012, *the effects of a culturally based math curriculum on Alaska Native students' academic performance.*

Findings

The results of the statistical analysis of the six distinct trials show that the Yup'ik students using the treatment curriculum outperformed their rural Yup'ik control counterparts at statistically significant levels. Interestingly, urban students and other ethnic groups using the treatment curriculum also showed statistically significant gain scores when compared to control groups.

			# of students	# of classes	Average Pre-Test %	Average Post-Test %	Average Gain Score	Std. Deviation of Gain Score	p-value for test of Gain Score
Fish Racks	Spring 2001	Treatment	51	4	30.21	37.19	6.98	13.69	0.015
		Control	27	3	32.45	31.10	-1.35	14.72	
	Spring 2002	Treatment	50	5	33.80	52.80	19.00	15.58	0.001
		Control	51	4	40.05	48.14	8.09	17.96	
	Fall 2002	Treatment	38	4	24.63	39.76	15.13	16.38	0.000
		Control	50	4	33.06	27.96	-5.10	9.58	
Egg Island	Spring 2002	Treatment	53	4	40.57	56.32	15.75	16.95	0.036
		Control	40	4	35.94	45.63	9.69	14.38	
	Fall 2002	Treatment	73	8	29.66	47.56	17.90	17.37	0.053
		Control	52	5	34.12	46.50	12.38	14.14	
Berry Picking	Spring 2003*	Treatment	53	6	30.23	48.38	18.15	22.28	0.001
		Control	32	3	42.83	47.10	4.27	14.45	

Table 1: Quantitative Data Results

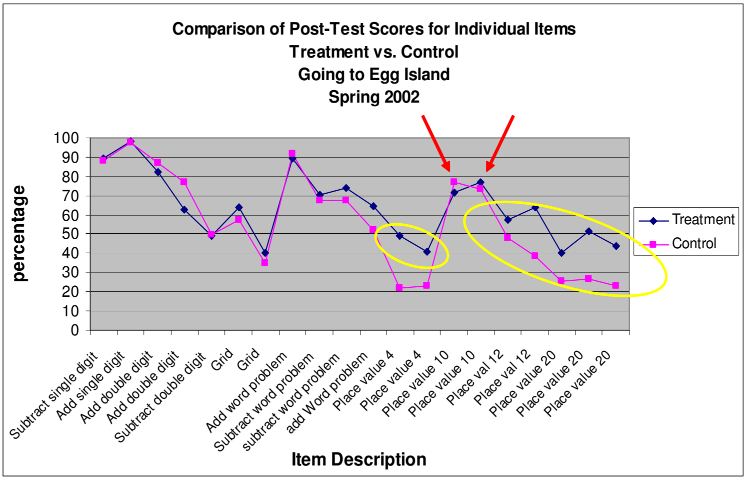
Table 1 summarizes the portion of the data collected for each of the six trials from rural students only. Note that during the spring 2003 trial (marked with an *) of the Berry Picking module the numbers were too small to separate the rural from the urban students. Thus, in that case, the reported values are for all treatment and all control students. The number of rural students, number of rural classrooms, average pre-test percentage scores, average post-test percentage scores, average gain in scores, standard deviation of the gain in scores, and the p-value are reported separately for treatment and control groups for each trial. In each trial, the difference in gain score between the treatment and control groups was statistically significant beyond the accepted standard of $p \leq 0.05$. Therefore, the treatment did, subject to the assumptions, improve students' math performance concerning the math topics tested. Note that the p-value relates to the level of statistical significance, so we found statistically significant results in favor of the treatment at the 98%, 99%, 99%, 96%, 95%, and 99% level, respectively.

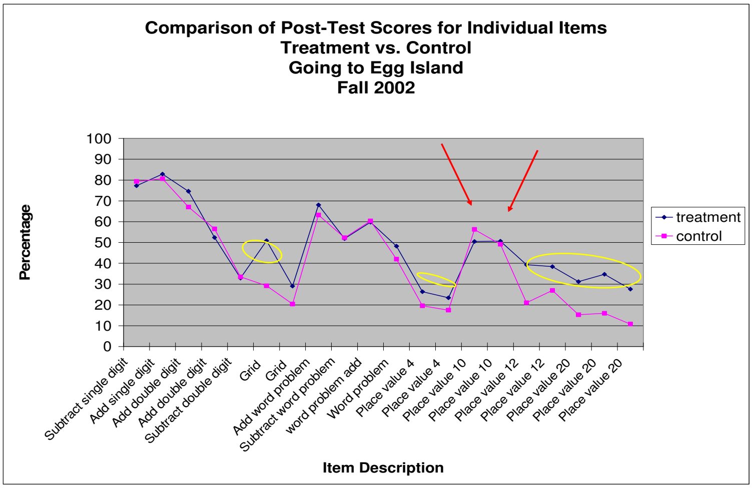
Locating the Effect

Since our data shows statistically significant gain scores for the treatment, we changed our focus to better understand the effect and to locate it.

Item Analysis for Going to Egg Island: Adventures in Grouping and Place Values

An item analysis enabled us to learn which items were easy and which were difficult, and to analyze differences across the items by treatment and control groups, locating the effect. The figures below show both item analyses for the 2nd grade.





This shows that for basic operations, word problems, and somewhat for place value or grouping in tens, there is virtually no difference between treatment and control. Grouping in 4s, 12s, and 20s generates most of the variance between the groups. Items pertaining to grids (reading and identifying coordinates) show some of the treatment effect. Further analysis of the data and comparisons between the treatment and control groups shows the following:

- Although the treatment curriculum does not teach base 10, students who engaged in the treatment hold or fall slightly behind the control group.
- Students in the treatment group appear to be able to generalize their thinking about grouping to groupings in other configurations than the typical school tasks of grouping in tens.

We also performed an interval analysis to further locate the effect by students' starting point (pre-test scores). Except for the lowest performing groups (interval analysis), all treatment groups outperformed all control groups. Lastly, we have conducted a treatment study to examine the actual implementation of the module. To date, this study is continuing and the predictive value of our scoring rubric is only low to moderate, with R-values ranging from approximately 0.3 to 0.5.

Qualitative Findings

The culturally-based math modules appear to provide access to school math for rural students and teachers. The locally familiar context and stories that accompany some of the modules, as well as the inclusion of local people who are known to the students, seem to connect them to the math (interviews with students in the village of Manokotak, Alaska, 2003). Further, some underlying values incorporated into the stories appear to motivate or increase students' interest (Frank Hendrickson's students, spring 2003). Students seem to like the use of Yup'ik games and hands-on projects (various student interviews). Students relate their own stories to those shared through the modules, for example: berry picking, fishing for salmon, building fish racks, and others.

Access and interest opens mathematical pathways not otherwise opened by such math curricula as *Saxon* or *Everyday Mathematics,* among others. Students across sites mention their connections to the underlying themes and subsistence activities of the module.

A novice Yup'ik teacher described her experience teaching the Fish Racks module as follows:

> Following the textbook [standard math curriculum] there's more worksheets, and you make copies, and you follow whatever the textbook asks you to do step by step. With fish rack it's kind of they're more into the concept and not trying to get so many problems . . . I was more open and observant with the fish racks than trying to take control like with the textbook, like I have to control everything and everything has to be, you know, in a certain procedure.

She states further:

> My students and I were comfortable. . . We had ownership of, you know, so I didn't have to introduce them to a whole lot of stuff. They knew all the stuff already and it was a good way to teach, or have them discover math concepts and strategies to find the solutions or whatever . . . it was more relevant to them; something that I know they're going to remember for the rest of their lives.

Yet, not all was positive in the use of the culturally-based curricula. In one example, students refused to use the module, stating to their teacher, "We already know about this" (paraphrased). A local Yup'ik teacher was asked to talk to that class. She shared with them, in Yup'ik, why she thought the curriculum was important for them. In another instance, a veteran teacher said that he expected the parents to be more interested in schooling because of the module, but they were not. Lastly, one boy, thought to be autistic, recently moved from a rural village to Anchorage and became interested in one of the project's story books. This eventually led to the student feeling more comfortable, becoming a class leader, and totally reversing the perception that the school had of him. Other teachers have reported less dramatic anecdotes of students who usually do not participate in the classroom sharing and being more involved when the CBC was used.

Concluding Remarks

This project has made a number of significant and practical steps towards redressing the colonial education in Alaska by:

- Incorporating Yup'ik elders' knowledge and ways of being into a culturally-based math curriculum.
- Finding statistically significant results for treatment students vs. control group students.
- Repeating results across three different treatment curricula, which lend even more credence to these results.
- An item analysis for one of the modules showing repeating patterns across the treatment and control groups; again lending additional credence to the plausibility that the treatment curriculum had a positive effect on students' performance.
- Closing the academic achievement gap between rural AN students and their urban counterparts.

The long-term and continuing involvement of the elders, who shared their knowledge and partook in the curriculum development process for more than a decade, speaks loudly to the importance of this topic to them (Frederick George and George Moses (elders), 2003 Summer Institute, Fairbanks, Alaska). Increasing student access to mathematics by framing and contextualizing the problem situations, the background for the math problem solving, and the challenging mathematics all appear to contribute to students' success. Yup'ik teachers state that they have had their eyes opened through this collaborative process of working with elders and math educators, which has positively affected their appreciation for their own cultural heritage and the role it can play in modern schooling. From the exclusion of all things Yup'ik to the slow inclusion of Yup'ik knowledge and ways of being is a major transformation.

However, the curriculum does not stand alone; it is an instrument in the hands of teachers. The interplay between curriculum, teacher knowledge, attitudes, experience, students, and the particular socio-historical relationship between school and community all affect the results. At this time, it is not possible to state what Yup'ik cultural factors, if any, are making the difference. A further unpacking of the complex factors that appear to be promoting students' increased math performance still remains.

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PILOTING THE SOFTWARE *SONAPOLYGONALS_1.0*: A DIDACTIC PROPOSAL FOR THE GCD

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In the paper we present a didactic unit designed within a research project on Arithmetics. The unit objective is to introduce the notion of the Greatest Common Divisor through sona, *sand drawings from African culture, and their representation by an appropriate software. A brief description of the project framework, the practice of the* sona *and the guidelines of the didactic proposal, as well as a sketch of the main characteristics of the* SonaPolygonals_1.0 *software are presented. The first findings of a pilot project at a few second grade lower seecondary schools in Italy will be also discussed in the DG15.*

The research framework

Our educational aim is to show how mathematics, hidden in traditional activities from non-western cultures [D'Ambrosio], can be actualized and become a tool to better vehiculate new concepts in math classes, while valuing different cultures in an intercultural approach to math education [Favilli], [Favilli, Oliveras & César], [Favilli, Maffei & Venturi]. In this way, we agree with Vithal&Skovmose's remark that, among the four main research strands in ethnomathematics they identified, the educational one is the natural and final convergence point of the other strands, which concern the history of mathematics, the mathematics of indigenous people and the mathematics of professionals.

The didactic goal of the project is to engage pupils in reflective activities. In fact, it is well known that the students' main complaint about mathematics is that it is too mechanical and inflexible. Students very often try to solve problems by only applying memorized rules, with poor understanding of the

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concept involved. We believe that a more attractive approach to some mathematical concepts could be a solution.

In view of that, a non-standard didactic proposal for the introduction of the Greatest Common Divisor between positive natural numbers has been developed, under the assumption that GCD is too often introduced by teachers in a technical and algorithmic way. Pupils, therefore, hardly realize the meaning and the potential of this concept, because GCD is usually associated just to fractions and their operations. The final result is poor attention and lack of interest which, in turn, cause hard comprehension of the concept.

To make this comprehension easier, the suggested didactic unit is founded on the *sona* (sand drawings from African tradition), the relationship *sona* – GCD and the implementation of a software that allows both to design such drawings and calculate the GCD of two positive natural numbers. Furthermore, in our proposal, pupils are given the opportunity to approach and construct the GCD concept, step by step, both working individually and collaboratively, linking geometrical aspects to arithmetic notions. The teacher's role is mainly to mediate the interactions among students and lead them gradually to institutionalize, as the final step, the GCD notion.

Many results of an Internet search about "Greatest Common Divisor" and related items, such as "Euclidean Algorithm" and its Extension or "Prime Factorization" and "Coprime Numbers", can easily show how hard is to find a different approach to the introduction and definition of these arithmetics concepts: the search gives clear evidence of the existence of much software implementing the different algorithms, but all of them seem to refer just to the arithmetic definitions.

The *sona* drawings tradition

The *sona* (singular: *lusona*) drawings (Fig. 1) practice belongs principally to the Tchockwe people, but also to other people in Eastern Angola.

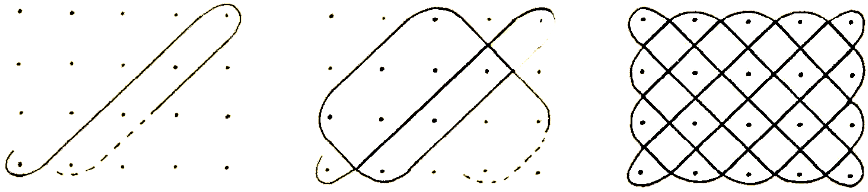

Fig. 1: ***The plaited-mat design: a class of sona deriving from mat weaving***

The story-tellers from these populations use to make these sand drawings to give a better and more attractive description of their stories. Young boys learn the meaning and the execution of the easiest sona *during the period of intensive schooling (the* mukanda *initiation rites).*Paulus Gerdes greatly contributed to the reconstruction of the mathematical concepts involved in these *sona*, which can be classified in different classes, according to the rules given to draw them. Gerdes described some didactic uses of these drawings as well and is still developing the research on their mathematical properties. One of the first properties shown was that the number of lines (polygonals) necessary to complete each given *sona*, in accordance with a short list of drawing rules, exactly corresponds to the GCD of the two positive numbers representing the *sona* dimensions.

The software *SonaPolygonals_1.0*

Considering the given drawing rules, we have implemented, using Java Programming Language, a graphical programme – *SonaPolygonals_1.0* – which draws, in movement, the *lusona* and computes the number N of lines necessary to complete a sona of PxQ points. The programme asks for inputs two positive natural numbers, P and Q, and the ordinate Y0 of the point which we decide to start drawing the *lusona* from. Y0 must be an odd number because the grid of PxQ points is inserted, to ease the procedurre, in a rectangle of dimensions 2Px2Q (Fig. 2).

The main procedure of the program contains the four different ways we have identified in which we can move inside the grid in order to draw the *sona*. We call evolution-types the possible segments that cut at 45º the side of the rectangle. *Propag* is the variable that tells us what kind of evolution we are making, hence its value varies from 1 to 4.

Going back to the main procedure, we see how N is calculated. At the beginning its value is 1. A counter tells us how many times the line, created in a continuous motion, reaches the y-axis (until, obviously, it rejoins the starting point of the line itself and then stops). Suppose we enclose with a single loop all the points of the *sona*; then, the value of the counter is Q and the procedure does not have to modify the value of N: it carries on being 1 and we have finished. On the contrary, if we need more than one loop to enclose all the dots, every time we begin to draw another line, the value of N is increased by 1.

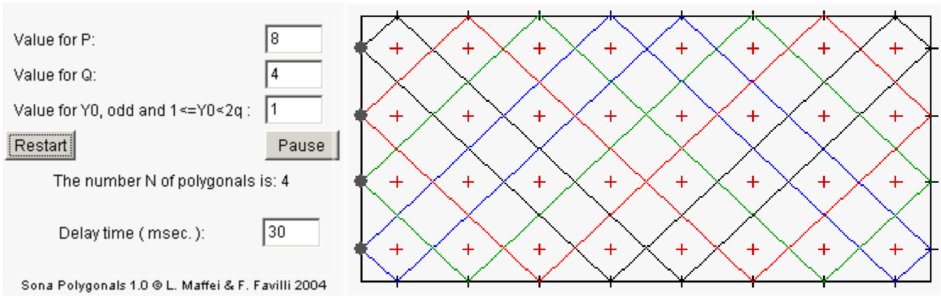

Fig. 2: ***On the left side window: the inputs and the numerical output; on the right side window: the graphical output – the*** **sona**

The structure of the didactic unit

The required prerequisites to carry out the unit are:

- Divisors and Multiples of a natural number;
- Divisibility rules;
- Factorization in prime numbers of a natural number.

The whole unit requires thirteen hours' lesson. This time is divided, in succession, into four activity lessons, each lasting for two hours (except for the second of only one hour), and two verifications, each lasting for two hours. Moreover, between the two verifications, the teacher spends a period of time (approximately three or four hours) explaining to pupils the *classical* methods for the computation of the GCD between positive natural numbers..

A rough description of each activity is given explaining the aims the unit has been conceived for and the methodology it should be developed with. An example of Exercise from each activity form is also shown to help the comprehension of the didactic proposal.

***Activity 1**: Let's learn to trace!*

Aims: *To understand the rules to trace a* lusona *and to realize that all the points are embraced by polygonals.*
Methodology: *Pupils trace, on their own, the lines necessary to complete a few sona: no indication is given to them (Fig. 3). Space is allowed to try to of discovering the rules the drawing obeys (each pupil keeps working on his own). A discussion is opened among the pupils, with the mediation of the teacher, to compare and contrast their findings. The activity ends with the formalization of the correct rules of the sona drawing.*
Exercise 1 – *Look at the figure and complete the tracing.*

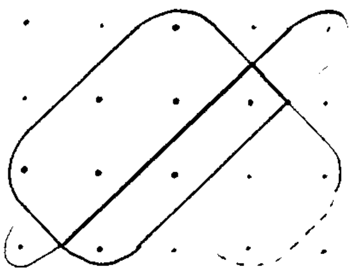

Fig. 3: ***An extract from the*** **Let's learn to trace!** ***activity form***

<u>Activity 2</u>: Let's continue to trace!

<u>Aims</u>: *To strengthen the rules to trace a* lusona *and to observe that the drawings considered in the first two activities consist of a single line.*
<u>Methodology</u>: *All the exercises are in charge of the pupils* (Fig. 4)*; the teacher, if necessary, controls that everyone reaches the aims.*
<u>Exercise 1</u> – *Try to trace a* lusona *in each of these grids having in mind the singled out rules.*

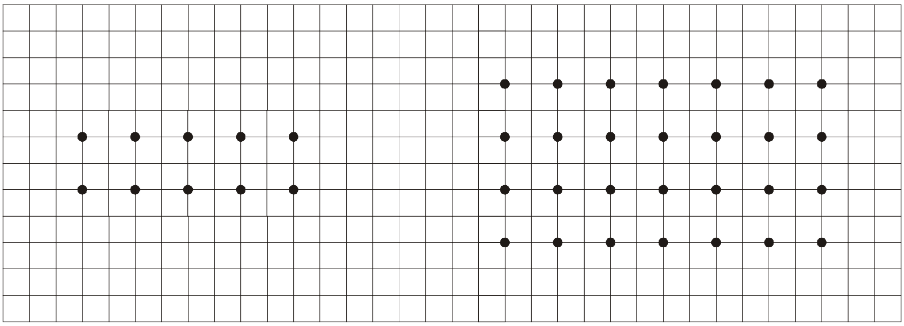

Fig. 4: ***An extract from the*** **Let's continue to trace!** ***activity form***

<u>Activity 3</u>: Let's count the lines!

<u>Aims</u>: *To consider a new class of* sona *which needs more than one polygonal to be drawn. This apparent break point should be overcome quite easily, paying attention to the remark made during the first activity: all the points of the* sona *must be embraced by lines.*
<u>Methodology</u>: *Pupils trace, on their own, the lines necessary to complete a few* sona *belonging to the new class* (Fig. 5)*. Pupils are asked to write the number of lines they have drawn for each* lusona*. A discussion should start*

among pupils, with the mediation of the teacher, to compare and contrast their results. To validate or correct the results obtained, each pupil uses the SonaPolygonals_1.0 *software. A bit of time is dedicated to show pupils how to interact with the software.*
Exercise 4 *–Try to trace a* lusona *in the cases (6,3) and (6,4), using, if necessary, different colours for different polygonals.*

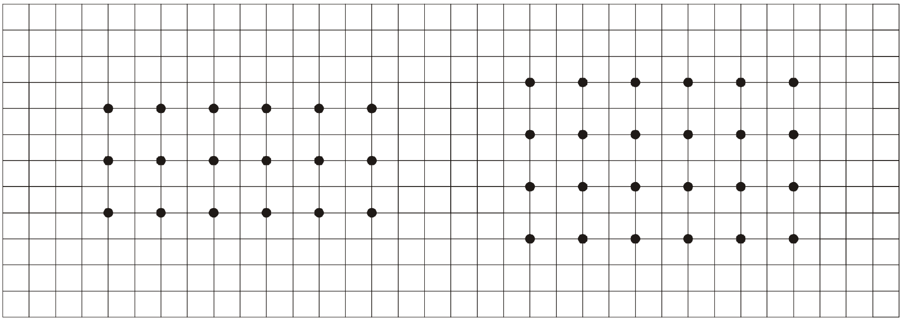

Fig. 5: ***An extract from the*** **Let's count the lines!** ***activity form***

Activity 4: The number of lines is...

Aims: *To identify the number of lines N necessary to draw a* lusona *of PxQ points with the GCD(P,Q).*
Methodology: *During the whole activity the software is available to pupils. They are helped in their construction of the concept, being asked, first, to enumerate the divisors of each number and, then, the common divisors of both positive natural numbers* (Fig. 6). *They should discover that N represents a particular common divisor: the greatest one! In the last part of the activity the teacher introduces the notion of Greatest Common Divisor.*
Exercise 7 – *Fill in the table referring to a few* sona *you have drawn.*

(P,Q)	**Divisors of P**	**Divisors of Q**	**Common Divisors**	**Number of lines**
(3,3)				
(9,3)				
(8,6)				
(18,12)				

Fig. 6: ***An extract from the*** **The number of lines is...** ***activity form***

Test 1 and Test 2:

When the four activities have been developed, a first test is submitted to the pupils to evaluate the level of comprehension both of the geometrical rules to draw a *lusona* and of the concept of GCD.
The second test is submitted soon after the introduction by the teacher of the usual methods to compute the GCD.

The current research

As for our research, we are interested in evaluating the findings of a pilot experience about the unit in a few schools. In this way we will be able to prove or disprove our conjecture about the introduction of GCD.

Some teachers in the same schools have been asked to co-operate to the project, although not completely involved in the didactic experiment. Their pupils, in the same grade, have been submitted the same second test as their peers in the piloting classes. It has been therefore possible to compare the achievements of the two sets of pupils and to better evaluate the real effectiveness of the proposed didactic unit.

Results from this comparison will be presented to the attention of the DG15 audience, with additional details on the didactic proposal and *SonaPolygonals_1.0* software.

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RICH TRANSITIONS FROM INDIGENOUS COUNTING SYSTEMS TO ENGLISH ARITHMETIC STRATEGIES: *IMPLICATIONS FOR MATHEMATICS EDUCATION IN PAPUA NEW GUINEA*

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In the context of current curriculum reform being implemented in the primary and secondary education sectors of the national education system of Papua New Guinea (PNG), this paper explores the possibility of utilizing and building on the rich cultural knowledge of counting and arithmetic strategies embedded in the country's 800-plus traditional counting systems. This is based on the commonly accepted educational assumption that learning of mathematics is more effective and meaningful if it begins from the more familiar mathematical practices found in the learner's own socio-cultural environment. Based on the basic number structures and operative patterns of the respective counting systems from selected language groups, the paper briefly describe how the rich diversity among these language groups can be used as the basis to teach basic English arithmetic strategies in both elementary and lower primary schools in Papua New Guinea.

Introduction

It is probably true to say that there is no country on earth having a staggering cultural diversity like Papua New Guinea (PNG), a nation with well over 800 distinct known languages. Politically, such diversity is a good catalyst for fragmentation of any nation, but after almost three decades of nationhood since September 1975, its citizens are proudly accepting such diversity as the strength upon which to build the future of their nation's political, economic and social systems (Clarkson & Kaleva, 1993; Kaleva, 1992; Matang, 1996). The current education reform being implemented in the country is a significant testimonial to the above belief and is mainly aimed at not only increasing accessibility to education services but also advocates a reform in

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the nationally prescribed school curriculum. Though such diversity may seem problematic in other areas of government, the need to develop a national education system that is both culturally relevant and inclusive reflecting the strength of its cultural diversity in its national curriculum, were factors that were considered much more fundamental and important. Subsequently, it is these principles that have formed the basis of the current education reform being implemented in PNG (Dept of Education, 2002; 2003). Moreover for almost three decades of nationhood, education has always been seen by both past and present leaders at different levels of government to be the single most important binding factor in maintaining national unity, an achievement not many other countries having similar cultural diversity have been able to do. It is with such background in mind that this discussion paper looks at the possibility of utilizing the rich cultural knowledge of mathematics or ethnomathematics in PNG as a means of bridging the gap between what school children already know about mathematics from their cultures and school mathematics. It is also a move taken to preserve the diversity of rich cultural knowledge, not only of mathematics, but also those indigenous knowledge systems that relate to other prescribed school subjects, much of which are orally kept and are fast disappearing (Kaleva, 1992; Lean, 1992; Matang, 2002).

From hindsight, if there is to be any school subject that is the last to talk about culture and its value systems, it has to be mathematics, particularly in the light of the current dominant view of the subject as being both *culture-* and *value-free* (Bishop, 1991; Ernest, 1991). Hence, the above dominant view has had greater negative educational impact on both the mathematics instruction in schools and the learning of school mathematics by school children in PNG. Given the country's staggering cultural diversity of 800-plus languages where both the mathematics teachers and the school students have very strong cultural ties with the value systems of their respective cultural groups, mathematics education in PNG has never been easy. In fact almost all national teachers who currently teach mathematics in the National Education System (NES) have been trained under the educational assumptions of the current dominant view where mathematics is seen as being independent of any cultural values, practices and its knowledge-base systems. The situation is no different for PNG school children, because much of the teaching of school mathematics does not take into account the rich mathematical experiences that school children bring into every mathematics classrooms. Hence, almost all PNG school students come to view school mathematics as being of no relevance to what they do to survive in their everyday life within their respective communities, further increasing the mathematics learning difficulties in schools (Bishop, 1991a; D'Ambrosio, 1991; Masingila, 1993; Matang, 1996; 2002). The struggling experiences that every school child goes through in learning school mathematics are best described by Bishop (1991b, p. *xi*) that,

> Mathematics is in the unenviable position of being simultaneously one of the most important school subjects for today's children to study and one of the least well understood. Its reputation is awe-inspiring. Everybody knows how important it is and everybody knows that they have to study it. But few people feel comfortable with it; so much so that it is socially quite acceptable in many countries to confess ignorance about it, to brag about one's competence at doing it, and even to claim that one is mathophobic!

Given the above situation, many research articles in both ethnomathematics and mathematics education (e.g. Masingila, 1993; Bishop, 1991a; 1991b; D'Ambrosio, 1990; 1991; Matang, 1996; Saxe, 1985) have identified *culture, language* and *learning modality* as the three most significant factors responsible for learning difficulties in school mathematics. Hence, this discussion paper aims to address the impact of the above three factors not as separate contributing factors to mathematics learning difficulties in schools, but to address them as relative factors. With such background in mind, the paper will specifically discuss how mathematically rich and meaningful counting structures embedded in almost all PNG traditional counting systems can be used to build on and link the teaching of formal English arithmetic strategies under the following sub-headings.

i. The diversity of PNG counting systems and its link to English arithmetic strategies.
ii. Implications for Mathematics Education.

The diversity among different PNG counting systems and their link to English arithmetic strategies

Like many other cultures around the world, the concept of number and its applications to all aspects of everyday life by members of a particular language group in PNG is rarely seen to be conducted in total isolation to every day cultural activities such as fishing, hunting, exchange ceremonies, etc. In essence, the development and use of some form of numeration system by any cultural group in PNG, irrespective of its primitiveness, is as old as the historical life of the cultural group itself. (Boyer, 1944; Lean, 1992; Smith, 1980). It is also important for the reader to take note that, like any spoken language which is not static, the fluidity in modifying the numeration system by the respective cultural groups to accommodate change in a cash economy like PNG (see Saxe & Esmonde 2001; Saxe, 1985; 1982a; 1982b) further gives rise to even greater diversity among these language groups and their respective counting systems.

Given the above background, the nature of diversity that exists between the different PNG traditional counting systems is by no means any different to the cultural diversity that exists between the 800-plus languages of PNG. Hence, the selected counting systems shown in Table 1 below gives the reader a very brief run down of the nature and the extent to which these diversities exist between different counting systems in terms of both the *frame words* used in constructing other numbers, and the *operative patterns* like 6 being *5+1* and 12 being *2 fives plus two* (Lean, 1992). The selected counting systems were chosen not only because they represent a range of commonly used counting systems in PNG, but also because they possess many important features of the majority of these counting systems by linking them meaningfully to the teaching and learning of English arithmetic strategies in schools.

Language Name	Counting System Features	Special features (Operative pattern) linking each system to English Arithmetic Strategies
Roro	10, 100 cycle system	Frame pattern: 1 to 5, 10, (40), 100 Operative pattern: 6=3x2, 7=3x2+1, 9=4x2+1, 12=1 ten + 2
Buin, Uisai	Cycle of 10, 100	7 is 3 before 10, 8 is 2 before 10, 9 is 1 before 10
Lindrou (Manus)	10, 100, 1000 cycle system	Frame pattern: 1 to 6, 10, 100, 1000 7=10-3, 8=10-2
Kate	2, 5, 20 cycle digit tally system	Frame pattern: 1, 2, 5, 20 Operative pattern: 3=2+1, 7=5+2, 8=5+2+1, 12=5+5+2, or 15=5+5+5=3x5
Gahuku (Goroka)	2, 5, 20 cycle system	Frame pattern: 1,2 Operative pattern: 3=2+1, 6 to 9 = 5+n
Hagen (Medlpa)	2, 4, 8, 10 or 2, 4, 5, 8 cycle system	Frame pattern: 1, 2, 3, 8, 10; 1, 2, 3, 5, 8 Operative pattern: 5 to 7=4+n, 6=5+1 or 4+2 or 4+3

Table 1 – Selected languages and structures of their counting systems

Though the majority of PNG counting systems have an operation pattern that is based on what Lean (1992) describes as digit-tally system (see Kate and Gahuku in Table 1), there are also other types of counting systems (e.g. Oksapmin) that are classified as body-part tally-system (see Saxe, 1981).

From Table 1, it is not difficult to identify the meaningful linkage that exists between the English arithmetic strategies taught in schools and the operative patterns of the respective counting systems as highlighted by the selected counting systems. For example, the operative pattern in Roro counting system for 7 is 3x2+1, in Buin counting system, 7 is 3 before 10, and in Kate counting system (the first author's counting system), 7 is same as 5+2. All these examples while equally correct in expressing 7, they are significant from the teaching point of view. Unlike the current English (Hindu-Arabic) counting system used in schools, these counting systems also provide the extra information on the relative number sequences in terms of their order of occurrences (Wright 1991a; 1991b). For example, 5, 6, 7, 8 in Kate is *memoc, memoc-o-moc, memoc-o-jajahec, memoc-o-jahec-o-moc* with morphemes that can assist children in easily remembering the order of numbers. This is because each Kate number word has meaningful linkage with their respective operative patterns of 5=5, 6=5+1, 7=5+2, and 8=5+2+1, an arithmetic strategy that is also important in addition of numbers as shown in Table 2 below).

The basic ideas of numeracy as embedded in the operative structures of each counting system can be extended further to cover other number concepts such as subtraction and counting in decades. For example, Buin and Lindrou languages in Table 1 have 7 as 3 before 10 and 10-3 or 8 as 2 before 10 and 10-2 using *ten* as the basis for counting numbers. In addition, this also helps with the representation of numbers and their relative positions on the real number line such as the one provided by the counting system of Buin (Uisai) language of Bougainville. It should be noted that, the ability of both the teacher and students to make mathematical inferences from basic number concepts such as those found in the respective counting systems in PNG is really the essence of what mathematics is all about. Hence, it only requires the teacher to recognize and take advantage of the children's very own traditional counting systems because they provide meaningful link between the basic concept of counting and the respective counting strategies as represented by the number words. As indicated in Table 2 below, these are important number properties, features that are necessary for an effective learning of English arithmetic strategies.

The Roro counting system according to Lean (1992) is a Motu-type Austronesian Counting System where counting in twos is a common feature of quantifying items during important ceremonies. Moreover since the Roro Counting System, like Buin and Lindrou (Manus), is a 10-cycle system, it is not difficult for children to develop formation of decades with the assistance of a teacher because of the similarities it shares with English counting systems. The multiplication of *one-* and *two-digit* numbers in Roro counting system makes it easy for children to relate multiplication in English counting systems because the construction of large numbers in Roro is dependent upon its operative pattern that utilizes the idea of multiplication by 2 and of decades.

On the other hand the counting system from the first author's language namely, the Kate language, is an example of a Non-Austronesian digit-tally system that has 2 as the primary cycle, 5 which is *one hand* as the secondary cycle, and 20 which is same as *one man* as the tertiary cycle (Smith, 1980; 1988; Lean, 1992). Hence as shown in Table 2, the use of numbers 1, 2, 5 and 20 as frame pattern numbers by school children during the addition of *one-* and *two-digit* numbers in Kate counting system is not only seen to be efficient but also very meaningful. This is because the spoken number words for large numbers not only enable children to establish meaningful relationships among individual numbers in terms of their uniqueness, but more importantly they reinforce the *concept of addition* as the *sum* of either two, three, or all four frame pattern numbers namely, 1, 2, 5, and 20.

English Numeral	**Spoken KATE Number Words**	**Operative Pattern**		**Spoken RORO Number Words**	**Operative Pattern**
4	jahec-o-jahec	2+2		bani	4
6	me-moc-o-moc	5+1		aba-ihau	2x3
7	me-moc-o -jajahec	5+2		aba-ihau hamomo	6+1 or 2x3+1
8	me-moc-o-jahec-o- moc	5+2+1		aba-bani	2x4
15	me-jajahec-o-me-moc me-jajahec-o-kike-moc	5+5+5 or 10+5		harauhaea ima	10+5
26	ngic-moc-o-me-moc-o-moc	20+5+1		harau rua abaihau	(10x2)+6
Note: In Kate, *moc* = one, *jajahec* = two, *me-moc* = five (hand), *ngic-moc* = 20 (one man)					

Table 2 – Spoken number words and Operative patterns for Kate and Roro counting systems

For example, the spoken Kate number word for numeral 8 is *memoc-o-jahec-o-moc* which in English translation means *one-hand plus fingers-two-and-one,* which is further more equivalent to the English operative pattern of *5+2+1*. Like many digit-tally counting systems in PNG, the use of frame pattern numbers 1, 2, 5 and 20 in Kate to construct larger numbers is a useful mental strategy for formal English arithmetic strategies taught in schools. It is also important to note that the visible absence of important operational and structural features in the English counting system makes learning even more difficult for a 4- or a 5-year old Papua New Guinean child enrolled for the first time in Elementary school learning to count in English, given that it is not their own mother tongue.

The above view is supported by the preliminary data from the first author's research conducted early this year (2004) assessing the early number knowledge of 5- to 6-year old elementary school children learning to count and perform simple addition and subtraction tasks in the local vernacular of Kate language. The preliminary data analysis indicates that when children were given the choice to give their answers either in Kate followed by English or vice versa on three counting tasks involving the actual counting and summing up of two groups of concrete objects, 100% of the children got both answers correct in Kate and English. On mental addition tasks involving use of concrete objects (e.g. 7+3), 73% of the children got their answers correct in both Kate and English. Although further research is required to determine the extent of the impact of learning to count in Kate language on English arithmetic strategies, the current preliminary data somewhat suggest that counting in local vernacular is an advantage as indicated by the high performance rate of children on different early number assessment tasks. One of the notable advantages observed by the first author during the research is the children's ability to quickly switch between English and Kate counting systems when responding to questions on simple addition and subtraction tasks. This observation indicates that children would generally resort to using either of the two counting systems that they found easier to use in comprehending and computing numerical tasks. Hence this observation further supports the aims and rationale for the current education reform in PNG that strongly encourages the use of Indigenous knowledge systems to teach mathematics in schools.

Implications for Mathematics Education

The acceptance and acknowledgment of the use of children's very own out-of-school mathematical experiences by teachers in teaching school mathematics has implications for both the mathematics curriculum and the way mathematics is taught in schools.

In terms of the implication for mathematics curriculum in PNG, it is pleasing to note that the current education reform which is also aimed at curriculum reform in all nationally prescribed school subjects, has at least taken the first important step in encouraging the use of various local vernaculars as medium of instruction during the first three years of elementary school. As a consequence, there are strong indications suggesting a move towards the teaching of mathematics that takes account of the country's rich cultural knowledge of mathematics with a number of teachers' guides and syllabuses already written and produced for distribution to elementary and primary schools in PNG. In a move aimed at ensuring a culturally relevant and inclusive mathematics curriculum, the current reform encourages greater freedom and flexibility among elementary school teachers to explore the everyday use of mathematics outside of the formal classroom as the basis to teach school mathematics. This is an encouraging development because it enables the school children to relate school mathematics to everyday experiences thus portraying mathematics as a user-friendly subject.

One of the significant stumbling blocks to successful implementation of any curriculum reform in PNG in the past has been the reluctance of teachers to accept new changes. According to Clarkson and Kaleva (1993) the reasons for much of these failures were mainly due to an unwillingness by many experienced teachers to change their ways of presenting mathematics lessons particularly when these classroom practices have become so routine for many of them. However in the interest of meaningful learning of school mathematics by students, it is suggested that public lectures and seminars be organised by Curriculum Development Division to educate parents and teachers on the aims of the curriculum reform. This is because at the end of the day it is the teacher's action or inaction in implementing any curriculum reform will determine the success and failure of any curriculum implementation process. Hence, the ability of an individual teacher to successfully implement any curriculum reform will depend on (a) the role of the teacher under the new curriculum reform process, (b) teacher beliefs and values about the new curriculum reform and (c) teacher background knowledge in mathematics (Matang, 1996). In other words, if teachers are to effectively utilize students' mathematical experiences gained from everyday encounters as the basis to teach school mathematics as proposed by this discussion paper, then they need to be made aware of their expected new role under the reform. In the context of what is proposed here, the role of the mathematics teacher will basically involve changing one's view from being an *authority* and *transmitter* of mathematical knowledge to that of a *facilitator* of the teaching-learning process (Matang, 1996; 1998). Secondly, if a teacher beliefs and values are not in line with the rationale for curriculum change then it can become a barrier to effective implementation process. Finally, in order for the teacher to fully execute his/her duties effectively, it requires a proper training program that gives opportunity for them to do an in-depth investigative study of the mathematics content knowledge. This is necessary to give them confidence to approach the teaching of mathematics in the immediate cultural context of school children as envisaged by this discussion paper.

Within the context of the actual classroom situation, it is envisaged that the culturally relevant curriculum will promote the role of students as equal partners of the teaching-learning process where they are encouraged to be active participants of information-sharing process rather than passive recipients of information transmission process. On the whole what is expected of a teacher to do under this approach is to firstly acknowledge school children as not empty cups that need to be filled up, but are unique social beings with the ability to think for themselves. Hence it is the responsibility of the teacher to create a learning environment that promotes meaningful and interactive mathematical discussions not only between teacher and students, but also between the students themselves. A useful linkage established by the teacher between children's very own mathematical experiences and the formal mathematics classroom promotes self-esteem and ownership of the knowledge by students thus giving them that extra reason to learn school mathematics. This can be achieved by encouraging the children to describe the use of school mathematics in community-related activities such as the use of the traditional counting systems in important cultural ceremonies and other everyday community activities. This includes having the children telling the rest of the class mathematically related family stories or identifying the different types of mathematical ideas being used in pictures cut from newspapers or drawn by the teacher and children themselves (Masingila, 1993; Matang, 2002; 2003; Owens, 2002; Owens & Matang, 2003).

Conclusion

It is obvious from above discussions that one of the significant ways to reduce mathematics learning difficulties among school students is to develop the mathematics curriculum that takes into account the rich out-of-school mathematical experiences that children bring into the formal classroom. The use of traditional counting systems in teaching the formal English arithmetic strategies in schools is one such example that provide meaningful and relevant learning experience for school children at the same time bridging the knowledge gap between school mathematics and the existing Indigenous knowledge-base systems found in the respective cultures. The current reform in nationally prescribed school curriculum strongly encourages the use of Indigenous knowledge-base systems in teaching respective school subjects. This approach would undoubtedly require the teacher to readjust his/her approach to mathematics teaching to accommodate out-of-school mathematical experiences of their students as means to not only ensuring mathematics learning is meaningful but also enabling students to relate what they learn in school into their everyday encounters in life making mathematics to be both culturally relevant and inclusive.

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ETHNOMATHEMATICAL STUDIES ON INDIGENOUS GAMES: EXAMPLES FROM SOUTHERN AFRICA

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Indigenous games are an integral component of the broader scope of indigenous knowledge systems. Such games and games in general are usually viewed from the narrow perspective of play, enjoyment and recreation. Even though these are very important, there is more to games than just the three aesthetic aspects. Analyses of games reveal complexities about games that are not usually considered, like, for example, the history and origins of the games, socio-cultural developments and contributions to societal and national activities, mathematical concepts associated with the games, general classroom related curriculum development possibilities and implications. This presentation reports on research on indigenous games that took place in the Limpopo and North West Provinces of South Africa and in the Nampula and Niassa Provinces of Mozambique. These studies aimed to explore indigenous games of String figures, Moruba and Tchadji (Mancala type games) and Morabaraba (a three-in-a-row type game) from a mathematical and educational point of view.

Introduction

The term ethnomathematics can be understood in two ways, which are in some ways related to each other. Ethnomathematics is in the first instance used to represent a field of mathematics which studies the different types of mathematics arising from different cultures, i.e., the way Gerdes (1989) and Ascher (1991) use the term. However, ethnomathematics has its implications for mathematics education (D'Ambrosio, 1985; Bishop, 1988). As Stieg Mellin-Olsen (1986) referred to it, "*If knowledge is related to culture by the processes which constitute knowledge - as Paulo Freire expresses it - this must have some implications for how we treat knowledge in the didactical processes of (mathematical) education*"(p.103).

There are other views of ethnomathematics. These views point out, in general, that ethnomathematics includes mathematics practiced, used or simply incorporated in the cultural practices or activities of different groups in society. The pedagogical implication of ethnomathematics is generally not stressed in these views. In fact, up to some years ago, Mathematics was generally assumed to be culture-free and value-free knowledge. For example,

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failure in relation to school mathematics was explained either in terms of the learners' cognitive attributes or in terms of the quality of the teaching they received. There were several attempts to make mathematics teaching more effectively satisfactory to the learners, with few long term benefits, but social and cultural issues in mathematics education were rarely considered (Bishop, 1993). Recently, mathematics educators, particularly in Africa, pointed out in their studies that there is a need for integrating mathematical traditions and practices of Africa into the school curriculum in order to improve the quality of mathematics education (Ale, 1989; Doumbia, 1989; Eshiwani, 1979).

Generally, games are part of human culture. In many societies, Indigenous games have been inevitably linked to the traditions of a cultural group, and they have been an expression of a local people, culture and social realities over a period of time. In this context, indigenous games can also be regarded as culturally specific (Mosimege, 2000), in which a culturally specific game is defined as "*an activity in which one or more people may be involved, following a set of rules, and the players engage in this activity to arrive at certain outcomes. The outcomes may be the completion of a particular configuration, or the winning of a game. The importance of the game with its social and cultural implication would then qualify this game to be a cultural game. Specific terminology and language used within different cultural groups further categorises this cultural game into a culturally specific game*" (p.31).

In some countries, particularly in Africa, they have been several attempts to go back to the tradition and try to understand and valorise cultural traditions. For example, as part of a process to revive the indigenous games, the South African Sports Commission (SASC), through the Indigenous Games Project, launched in early 2001 the indigenous games of South Africa at the Basotho Cultural Village in the Free State Province. The launch concentrated on 7 of the 23 identified indigenous games in the nine South African provinces, which were regarded as most common throughout the country. Among them were games of string figures, three-in-a-row type games (a variant called *Morabaraba*) and Mancala type games (a variante called *Moruba*). As SASC (2001) refers, "*Over millenniums under the African sun, a rich heritage of play activities and games gave a kaleidoscope of cultures, environments, social and historical circumstances or as human expression of generations of our rainbow nation. Ranging from informal play-like activities whereby skills are improvised and rules negotiated to more formal games in which skills, rules and strategies became more formalized and extrinsically rewarded. A diverse of physical culture was created* "(p. 3)

In Mozambique, within the context of the Ethnomathematics Research Centre (MERC), in early 1990s, the exploration of mathematical and educational potential of indigenous games was identified as one of research area. In particular, the following games were identified for exploration: games involving string figures, games of the three-in-a-row type (a variant called *Muravarava*) and Mancala type games (variants called *N'tchuva* and *Tchadji*).

With regard to *N'tchuva*, a Mancala type game played in Mozambican provinces of Maputo and Gaza, an article appeared in the newspaper ('Notícias', 26.10.1976), entitled "*N'tchuva in our schools?*", proposing the introduction of this game in schools instead of Chess, claiming that "*N'tchuva, this game of holes and stones, is a game which can not only develop deductive thinking but also quick thinking ability. So, why not consider introducing this game in our schools as part of recreational activity through championships and tournaments?*" Much later in another short article in a Sports newspaper ('Desafio', 10.04.1995), which included a photograph of a group of young people playing *N'tchuva*, the need to disseminate this cultural practice among young people was expressed, since they really felt interested in these games.

With such a challenge in mind and within the context of ethnomathematics, the authors of this paper conducted two independent research studies in their respective countries, namely in Limpopo and North West Provinces of South Africa and in the Nampula and Niassa Provinces of Mozambique (Mosimege, 2000 & Ismael, 2002). The studies aimed at exploring the mathematical and educational potential of selected indigenous games. This paper discusses some of the findings of these studies.

Indigenuous games in Southern Africa

Malepa: String Figure Games

The history of the record of string figures in Africa dates back to almost 100 years (Lindblom, 1930). A pioneering work is found in Alfred Haddon's treatise of 1906 containing some ten references to the pastime from Negro tribes, most of them from Africa (Rishbeth, 1999). Most of the record suggests that either some anthropologists or some travellers who obtained the figures from an informant brought the string figures away from its indigenous place. For instance, Oxton (1995) mentions that string figures are still quite popular among Eastern Islanders, and oftentimes a traditional chant is recited while the figure is being made. This suggests that for the Eastern Islanders, making string figures is not only an activity to be engaged in but has other

social as well as cultural implications if the traditional songs that accompany the making of string figures are considered and analysed.
There are a variety of String Figure Games that are played throughout South Africa. The names used for the game and the different forms of the game vary from one place to another.

The String Figure Games that are invariably played are: String Figure Gates ranging from Gate 1 to Gate 7 and more gates; Cup and Saucer, *Monatlana* (known as *Menoto ya dikoko in Setswana*), Magic (disentangling the string around the mouth), Saw (usually demonstrated by two learners working together).

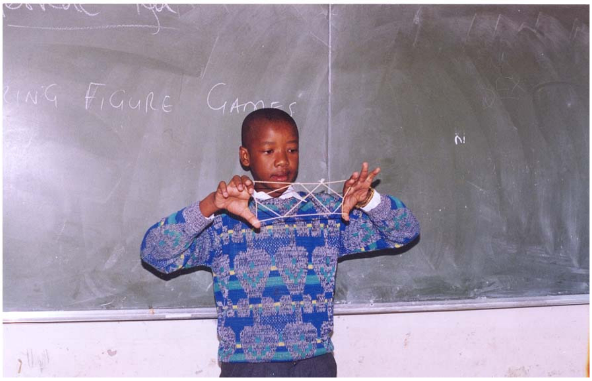

Fig. 1: ***A Learner Demonstrating Gate 2***

String figure gates are a specific kind of string figures. Some of their unique features that differentiate them from other string figures are:

a) They are started from the simplest gate to the most complex, and at times expertise is determined by the most complex or the highest number of gates that a person can make;
b) Other gates may be generated from those of the simpler forms. For instance, Gate 4 may be generated from Gate 2 by continuously adding 2 gates through a specific procedure;
c) Different instructions may be used for making the same Gate, and at times competition in making the gates may be based on the different methods of making the same gates that a player knows.

Morabaraba or *Muravarava*: Three-in-a-row type game

Morabaraba or *Umlabalaba*, as it is know in South Africa in *Sesotho* or *Xhosa* languages, or *Muravarava*, as it is known in *Ronga* language of the

Southern Mozambican provinces of Maputo and Gaza, belongs to the class of three-in-a-row board games. The variety of these games are known as Three, Five, Six, Nine or Twelve Men's Morris, according to the number of counters used for a specific version (Zaslavsky, 1973; Bell & Cornelius, 1988; Hopson et al, 1996).

In spite of the origin of three-in-a-row type games seemed to be traceable to the African continent, it is not clear on the exact origin and time of this game. However, there is no doubt that it has been in existence in African communities over a long time. There are references to Egyptian connection of these type of games (Zaslavsky, 1996; Gasser, 1996). These type of games is played in various parts of the world. About this, Zaslavsky (1996) argues that the game has reached many parts of the world through ancient Greek scholars who spent years in Egypt, well known as a centre of culture and learning from whom the Romans possibly learnt it and spread it as they continued to conquer Europe, the Middle East, and North Africa.

For the three-in-a-row type games there is essentially one principle, which is that of making a row of three counters. The object of the game is to make a row of three counters of the same colour, and to block the other player from making a row. A row can be made horizontally, vertically or along a diagonal (Zaslavsky, 1993).

The *Morabaraba* game is played by two players over three stages: In the first stage each player starts with 12 tokens, and the players take turns placing the tokens on the board. Every time a three-in-a-row is made, the token belonging to the opponent is taken. In stage 2 when all the tokens have been placed on the board, they are then moved from one junction to another with the aim of making a row of three. In stage 3 when all but three tokens are lost, the player concerned may "fly" a token to any vacant junction on the board with each move. The game is won when the opponent cannot move any of the tokens, or when an opponent has lost all but two tokens (See Fig. 2).

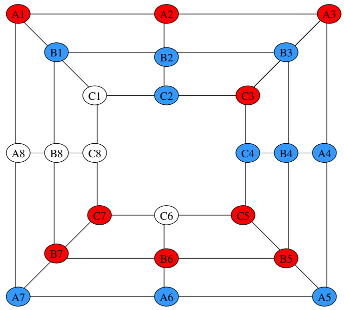

Fig. 2: ***Final Configuration of counters at the end of the placements on a Morabaraba Board (South Africa)***

Moruba and *Tchadji*: Mancala type games

Mancala is the generic name, given by anthropologists to a class of board games played throughout the African continent as well as in many other parts of the world. The word Mancala comes from the Arabic word ***manqala***, or ***ninqala***, from the verb ***naqala***, which means 'transfer' or 'move' things about (Murray, 1952 & Russ, 1984). These games are thousands of years old, and their origins are lost in antiquity and that they have originated either in Asia or Africa (De Voogt, 1997; Russ, 1984; Silva: 1994 & 1995).

Mancala games have only in the eighties received due attention as a research topic and have been studied from many different perspectives. There have been within the study of Mancala itself earlier general studies (e.g. Murray, 1952; Béart, 1955; Bell, 1960), which have been used as introductions to the more specialised recent studies written from art-historical, psychological, pedagogical and anthropological perspectives and on the use of Artificial Intelligence (Reysset, 1995). In the last twenty years Mancala games have been the subject of three doctoral theses (Townshend 1986, Walker 1990, De Voogt, 1995) and other extensive studies (Retschitzki, 1990; Silva, 1994 & 1995 and Russ, 1984). Townshend's (1986) anthropological research, studies the roles that games play in various cultures taking the example of *Bao*, a game of Mancala type played in East Africa. Retschitzki (1990) studied African children's development as measured by their skill in *Awélé*, a West-African game classified as being of Mancala type and also known as *Wari*.

More recently, De Voogt (1995) studied the limitations of human memory skill in the game of *Bao*, as played in Zanzibar Island. *Bao* itself has provided other earlier anthropological studies of symbolism in East-African Culture (See, e.g., Townshend, 1986).

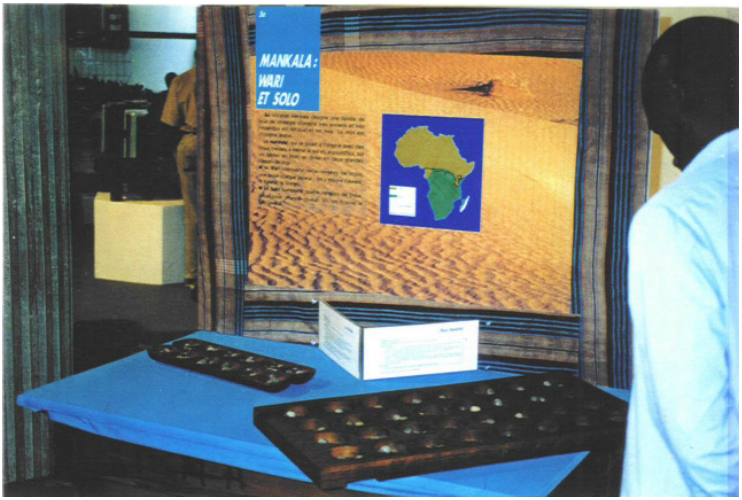

Fig. 3: ***Mancala type games:*** **Wari** ***and*** **Solo** [1]

Essentially, the playing of Mancala games (Fig. 3) consists of one basic movement, which involves ‘sowing’ and ‘capturing’ playing pieces using a board comprising rows of holes. Therefore, Mancala games are know as a ‘count and capture’ boardgames. They are generally played by two people or, on rare occasions, by two teams, on a board containing two, three, or four parallel rows of cuplike depressions or holes.

It is played with a predetermined number of identical counters. The goal is to immobilize or annihilate the other player(s) by the capture of the majority of seeds (Walker, 1990). Each move in a Mancala game causes multiple changes in the position of the pieces on the board. If multiple sowings and captures are allowed, these changes become even more difficult to calculate. In fact, the art of these games is the calculation of multiple changes, a prerequisite for all good players since strategies can only be executed once this skill has been acquired.

Moruba is a Mancala type board game that is popular in the Limpopo Province of South Africa. However, it is also known in other parts of the

[1] These boards were exhibited at the Pedagogical, Interactive and Intercultural exibition for ‘Jeux, Mathématiques et Sociétés’, which took place at the French Cultural Center in Abidjan (Ivory Coast), in November 1994

country. The version most common in the Limpopo Province is the 4 x 36 board, i.e., a board of 4 rows of 36 holes each.

Tchadji is another Mancala type board game played at Ilha de Moçambique, a small island in the Northern Mozambican province of Nampula. The playing of *Tchadji* at the Island has a long tradition. Presently, the Island is subdivided into small neighborhoods. The playing of *Tchadji* is organized in circles of players and many neighborhoods have their own group of players. *Tchadji* is usually played on wooden boards, which have four rows of eight holes each (4x8 board), carved into them. The pieces are fruit stones from *Caesalpinia Bonduc* bush. *Tchadji* is played by two people, only men, facing each other (Junod, 1962).

Research findings

In order to be able to explore the mathematical and educational potential of indigenous games the authors used a number of research strategies. In some instances, it was necessary to go back to natural settings of playing of a game and so ethnographic research by means of field work have been used. Because the game involve a number of mental operations it was necessary to interview the players of the games (*Tchadji & Morabaraba*). The educational exploration has been basically conducted in mathematics classrooms. In some stage, classroom interventions have been used (The probability approach based on indigenous games & the case of *Malepa & Morabaraba*). A number of interactions in the classroom (Free Play, Demonstrations by the Learners, Demonstrations by the Researcher, Interaction in the Worksheets and Questionnaires) have been observed, recorded and analysed (The case of probability approach based on indigenous games and the case of *Malepa & Morabaraba*). Also, pre- and post-tests have been used (The case of probability approach based on indigenous games). This section reports on some of the findings.

Indigenous games related mathematical ideas

In Malepa

The analysis of the *malepa* reveals a number of mathematical concepts. It is important to indicate that the list of mathematical concepts specified below is not exhaustive. It is possible that other mathematical concepts may be found through further analysis of the game and in the literature on string figures. In order for the following mathematical concepts to be easily identifiable from the figures that are made, a player needs to start by making any of the gates, preferably two or more. The following mathematical concepts are found in the analysis of string figure gates:

(i) Identification of a variety of geometric figures after making the different string figure gates: triangles; quadrilaterals (depending on how the string is stretched, quadrilaterals also specified into squares and rectangles);

(ii) Specification of relationships between various figures and generalisations drawn from these relationships: triangles and quadrilaterals: $y = 2x + 2$, quadrilaterals and intersecting points: $y = 3x + 1$, quadrilaterals and the number of spaces (spaces is given by the combination of triangles and quadrilaterals): $y = 3x + 2$

(iii) Symmetry: Symmetry in terms of performance of some steps in making the gates - performing an activity on one side similar to the one done on the other side; Exploration of the different types of symmetries and the related operations in the different gates - bilateral (reflectional) symmetry, rotational symmetry (different folds), radial symmetry, translational (repetitive) symmetry, antisymmetry; Various properties of symmetries; Disentangling the string along a specific line of symmetry which ensures that the string does not get entangled.

In Morabaraba game

The following mathematical concepts are found in the analysis of *Morabaraba*:

(i) Identification of various quadrilaterals (squares) and the similarities and differences between them.

(ii) Ratio and proportion between the lines and the squares making the complete *Morabaraba* board.

(iii) Symmetry: The various sides of the board; within each side of the board; Placement of tokens and repetitive movements of the tokens on the board.

(iv) Logical deductions in the execution of the various steps of the game.

(v) Counting

In Tchadji game

The ethnographic research and the subsequent analysis of *Tchadji* revealed some related mathematical ideas and processes. Since the *Tchadji*-board consisted of 32 holes containing a total of 64 seeds at the start of a match and since the game involves spreading seeds around the row of holes in an anticlockwise direction, it seems that some kind of counting has to take place when the seeds are distributed or even before. However, it has not been clear what is counted, when and how. The holes may be counted and even the seeds to be distributed to the holes may be counted.

The playing of *Tchadji* involves simple and complex moves. For instance, if a move takes more than a complete round, i.e., if the process of spreading seeds covers more than 16 holes in a player's rows, then it might be difficult to predetermine its outcome. On the other hand, a *Tchadji*-game, as played by the good players, is a very quick game. The players are able to perform more than 15 moves in about 90 seconds. Clearly, if mental counting takes place, then it must be performed very quickly.

Simple counting takes place in several moments of the game. With two seeds in a hole a player can advance two holes if it is not further connection with other holes ahead. In this case there are more than two seeds in a hole where a move is to be made, then several questions involving counting arise. How many seeds are in the hole? How many holes need to be covered in order for a player to be able to capture seeds, safeguard his seeds from being captured in a next move, empty a hole, or connect with a non-empty hole in order to advance further? For example, in the following situation (See Fig. 4), player A, at his turn, can capture seeds in column h (h3 & h4), if he plays in a2

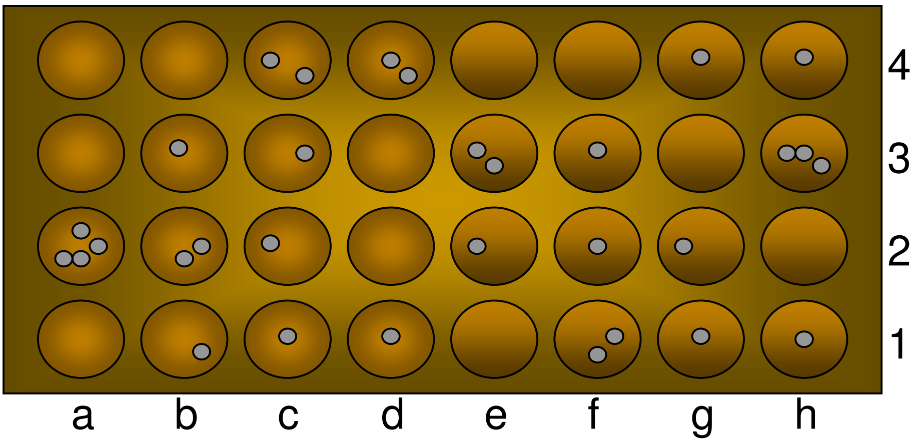

Fig. 4: ***Counting***

There he would be doing some counting. The 4 seeds in hole a2 only would not be enough to reach hole h2, but the 4 seeds in hole a2 with the seeds in the connecting holes d1 (1 seed) and f1 (2 seeds), i.e., with a total of 7 seeds in movement, leaving 2 empty holes, would cover the 9 holes from a2 to h2. Thus, counting is taking place here.

Also, a connecting operation is taking place here. The nature of moving seeds around the *Tchadji*-board, is such that this connecting operation cannot be classified solely as an addition of seeds. However, it is clear that the objective is to cover a certain number of holes in order to advance around the board. To accomplish this, certain number of seeds is needed. Thus, a kind of addition is taking place. If there were 9 seeds in a hole, it would be simple to advance 9 holes ahead (9 seeds cover 9 holes). However, it is rare to have this number of seeds in one hole. For some one experienced in playing *Tchadji*, a master player for example, the operations are performed extremely quickly. The players must have memorized the game and its structure, in such a way that facilitates the quick moving and thinking that precedes a move.

Probability aspects are equally related to the playing of *Tchadji*, for example, the chances of each player in winning the game. When playing Mancala there is an amount of uncertainty involved. Also, Eigen & Winkler (1975) referring to deterministic games, such as chess, argued that these games, because of their complexity, are uncertain with regard to the final result. For example, one would guess that a certain player should win. In fact, it can happen that he loses. A question that could be asked here is the following: 'What is for that person the basis for guessing that a certain player wins?' This person must be in possession of information that could make him decide to guess as he did. The person guessing would probably be thinking that this player is more likely to win and not that this player certainly wins. Thus, it is clear that in a *Tchadji* game there is a degree of uncertainty, which cannot be quantified in the classical way, but it can foster and develop certain aspects of probabilistic thinking.

Tchadji is also related to rotational symmetry (different folds) of the movement of the stones and to reflectional symmetry, based on the part of the board that each player uses to play. It also related to the concept of infinity, since there are certain moves in game that once started never end (Ismael, 2002).

Observations from the use of *malepa* and *morabaraba* in mathematics classrooms

Using culturally specific games in classrooms shows that the learners bring to class various levels of knowledge of the games. These range from a complete lack of knowledge to the ability to perform a variety of intricate manipulations with the strings. When afforded an opportunity to explore and share their knowledge, spontaneous groups get formed, groups in which learners who know some string activities play a leading role. Although this study did not necessarily isolate gender as an important factor to investigate, female learners seemed to be more conspicuous in engaging in string activities as most of them were selected to give demonstrations. Despite the instructions given to the learners on how to interact in some of the activities pertaining to the worksheets, the learners do not necessarily follow these as they arrive at specific activities more on their own than as a team working together.

For example, during the period of Free Play, different learners became engaged in a variety of string activities that they knew and able to exhibit in the first few minutes after each learner was given a string. As the learners became engaged in these activities, it was possible to notice that learners did not have the same levels of knowledge of activities in strings. A further analysis of the involvement of learners in these activities led to the identification of five distinct groups: (i) Learners who did not know any String Figure Activity (String Figure Gates or String Figure Configurations). (ii) Learners who did not know String Figure Gates but other String Figure Constructions such as the Saw, Cup and Saucer or even Magic i.e. disentangling of the string around a mouth. (iii) Learners who knew only one String Figure Gate. (iv) Learners who were able to construct a maximum of two String Figure Gates. This group also includes learners who could use more than one method to make the same gate and these methods differ from those used by most of the learners or the standard method for constructing the gate. (v) Learners who can construct more than two gates and other String Figure Configurations. They were also able to generate other String Gates from those already made. The numbers here are extremely low, at most two learners in each school.

The different levels of knowledge of string figure activities resulted in learners moving from their seating positions to nearer those who knew different string activities. The researcher classified this as spontaneous interaction among the learners as those who knew very little or knew nothing approached those who knew some of the activities and request them to show them (those who don't know) how the activities are done.

In the selection of learners to give demonstrations no preference was given to either males or females but to anyone who could make any kind of String Figure Construction, the majority of those who were selected to give demonstrations were females (51 out of 70 pupils). This does not necessarily reflect that the male learners did not know a variety of String Activities although this is also possible, but it may also have to do with the fact that the female learners were more forthcoming and more actively involved in the activities than the male learners. They (female learners) also seemed to be more interested in learning Gates if they did not know them before than most of the male learners were. The video recording of some of the shows some of the male learners engaging more in activities that had nothing to do with any of the String Activities that were envisaged but more an attitude that they did not know any String Activity but also did not seem to care to want to know.

One of the instructions that was specified to the learners in terms of interacting in the worksheets was that they are expected to work together on a worksheet and that they must actually discuss among themselves as they engaged in the different activities. This, however, did not always happen. On a number of occasions the following was observed:

(i) Some of the learners arrived at the final step of making a particular Gate when the other learners were still struggling with other steps. This does not refer to a situation in which some of the groups explained to me that one particular learner had serious difficulty in terms of the manipulation of the string in some of the steps and they had no choice but to abandon the learner, as they would never make any progress with the worksheet.

(ii) Some of the groups spent much longer time on a string activity in an attempt for all of them to reach the end of a specific gate together, as a result they would end up doing very little of the other requirements on the worksheet.

The learners who had given demonstrations in class tended to dominate in their groups in any or a combination of the following ways: (i) Reading through instructions, (ii) Answering the questions on the worksheet, (iii) Making the necessary gate or telling others how to make the gates. It also occurred that in some groups where one of the learners had given a demonstration and the questions required them to make a gate already demonstrated or the one that this learner was familiar with, the learners did not follow the instructions for making the gate as prescribed in the worksheet but would base their work on the knowledge of the learner knowledgeable in the gate. It also became very clear that learners tended to struggle to follow the instructions for making gates when written in the worksheet as opposed to watching and imitating a demonstration that is being given, whether by the learner or by the researcher. This realisation seems to suggest that the most successful way of teaching others how to make String Figure Gates is through demonstrating to them rather than giving them instructions to follow. Finally, availability of standard instructions does not necessarily make it easier for the learners to make any of the Gates.

From the probability teaching approach based on indigenous games

Attitudes towards mathematics

The use of probability teaching approach based on indigenous games in Nampula and Niassa provinces of Mozambique showed that very small changes were observed in attitudes towards mathematics between pre- and post-administration of the questionnaire in both experimental and control groups. A slight positive increase in the means measures was observed in the responses from both groups. However, in the pre- and post-questionnaire measures of both groups no statistically significant differences were observed between the groups. An interesting feature is the fact that the overall average on the post-questionnaire was greater than 3, i.e., above average. In the pre-questionnaire, the means were also around 3.

Nevertheless, the students of the experimental groups might have changed their opinion since the overall mean correlation between the pre- and the post-attitudinal questionnaire was pretty high, whereas in the control group was low. This could be an indication that the students from the experimental group might have changed their attitudes, perhaps for the positive way, when confronted with the game-approach for learning probability. The fact that there were no statistically significant differences observed in the overall means in the case of both questionnaires, does not mean that both groups responded to the questionnaires in the same way.

In fact, when asked how their feelings are with regard to their experience in learning with use of game the students responded generally that they enjoyed the lessons very much, they had fun in playing the games and analyzing the issues raised in the lessons, e.g., a student commented as follow: "*I did not imagine playing Tchadji in the classroom. I knew the game itself is not strange for me. It was strange to have seen it in the classroom. This is an experience that I never had before*". Another student said,"*the last sessions were very nice. The game practice was very nice. We used to play* *this game at home without knowing what is essential in it*". The facility of the student with *Tchadji* might have improved because of these lessons.

It became also clear that such type of lessons is never implemented at all. For example, a student commented as follow: "*I learned a lot (...) it was my first time to have lessons like that, I gained a lot of experience with the examples given here, they were practical lessons. Regarding other lessons I had, they were never given this way*". Another student expressed the same idea when saying, "*I liked the lessons, they very exciting because we were taught by doing...With this way of teaching you can learn really (…) other teachers should also teach us in this way if there is a possibility*".

Enjoyment and fun play an important role in learning mathematics and are some indicators of attitudes (Oldfield, 1991; Ernest, 1994). This finding suggests that the impact of this intervention on attitudes and on motivation was considerable and that the use of games can increase students' enthusiasm, excitement, interest, satisfaction and continuing motivation by requiring the students to be actively involved in learning (Klein & Freitag, 1991; Mcleod, 1992)

Performance in probability

The mean performance score on probability for subjects taught trough the game approach (Niassa) was 11.55 (SD = 2.45), and the mean performance for the subjects of the non-game approach (Nampula) was 9.21 (SD = 2.83) (See table 3). When the treatment groups were compared, a statistically significant difference was found in their performance measures, $F (1, 159) = 23.850$, $p < 0.0005$. For the purpose of adjusting for initial differences the pre-test measure was used as covariate (Pedhazur & Schmelkin, 1991).

The overall mean scores for two treatments in the pre-test on general mathematics turned out to be almost the same. However, the overall mean score of the control group on the probability test (post-test) revealed to be significantly less than the overall mean score of the experimental group. Thus, this difference, about 2.34, cannot be attributed to the initial group differences. The difference could be the result of using the game approach as a teaching strategy. However, there may be other acting variables, which may explain the obtained differences. In general, the students of the control group treatment were not able to explain properly the meaning of the concepts, whereas the students of the experimental group used examples experienced in the classes to explain their view, e.g., *If two master players are playing Tchadji, we can never know previously who is going to win* or *we do not have* *absolute certainty of what is going to happen*. In order to explain their thinking they also used examples of other games, e.g., *For example, for a soccer match we can never know beforehand how many goals will be scored.* This result suggests that using such games in the mathematics classroom is suitable for improving students' performance in mathematics, because the students make practice more effective and become active in the learning process.

Concluding remarks

These studies were directed to explore mathematical ideas involved in indigenous games and in the procedures for playing the games. This is one of the major research directions in ethnomathematics. The results indicated that, for example, *Tchadji*-players do count, think and act logically, do calculations, predict moves, visualise situations, recognise different numerical patterns while playing *Tchadji*. Therefore, it can be concluded that there is mathematical thinking involved in the playing of *Tchadji*. This mathematical thinking appears in the form of mathematical concepts, processes and principles and is not easily recognisable at all. Thus, this research has contributed to the field in describing the processes involved in this mathematical thinking. *Malepa*: the string figure game showed its relationship to some mathematical ideas like quadrilaterals and intersecting points, combination of triangles and different forms of Symmetry. It can also be concluded that playing *Tchadji* is a recreational activity and has a long tradition. The players learn to play *Tchadji* by watching other people play and then by playing themselves. In playing *Tchadji*, different strategies and tactics are employed in trying to secure a victory. The knowledge about these aspects of the activity of *Tchadji*-playing enriched this study.

Indigenous games relate directly to some of the Specific Outcomes in Curriculum 2005, the post-apartheid curriculum orientation in South Africa. In particular, Specific Outcomes 3 and 8 explicitly state aspects that relate to culture. Questions that have arisen since the advent of Curriculum 2005 have focussed on how to find specific examples that relate to the outcomes in the classrooms. Results of this study show that examples abound outside the classroom, examples that learners are to a greater extent familiar with. These examples can be used extensively to illustrate various aspects of Curriculum 2005.

The nature of the construct of indigenous games suggest that somebody needs to analyze the learners' activities to reveal related mathematical concepts and processes in their activities. Teachers are appropriately placed with their mathematical knowledge to translate the learners' knowledge into meaningful mathematical explorations. This is only possible when a platform is created in which learners' knowledge of activities outside the classroom is used to relate to mathematical knowledge in the classroom.

Besides the fact that they might be other confounding variables in the experiment study reported above, this particular study has showed that indigenous traditional games are so powerful as other types of western games. These games can enhance attitudes towards mathematics by creating fun and enjoyment for the students. They can also increase cognitive learning capabilities in mathematics. The potentialities of non-western games have not been explored so far and have not been documented at all. This study emphasized the need for explore this point and has contributed to this debate.

Particularly, countries with lack of resources for teaching can benefit from their own local and cultural resources. It is indeed a need for potential and for abilities for appropriate exploration of local and cultural resources and traditional games is an example of this kind of resources.

There is a great potential to use indigenous games in the mathematics classrooms. However, there are necessary imperatives to ensure that this is a success, as their use may convey negative connotations when these imperatives are not taken into account. The necessary aspects are:

(i) Indigenous games can also be analysed (mathematised) to reveal a variety of mathematical concepts that are useful in school mathematics.

(ii) When these games are analysed, the related historical, social, and cultural meanings and implications should be taken into account. Failure to consider the implications of these can lead to misconceptions and misunderstanding in the use of such games and as a result detract rather than enhance mathematical understanding. In other words, indigenous games should be considered in their entire context (historical, social and cultural), which is possible to find and use appropriately.

(iii) It is indeed true that games serve a variety of affective objectives when used in mathematics classrooms. However, games also serve a variety of cognitive objectives.

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ETHNOMATHEMATICS AND THE TEACHING AND LEARNING MATHEMATICS FROM A MULTICULTURAL PERSPECTIVE[i]

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A basic tenant of ethnomathematics is the sincere belief that all peoples use mathematics in their daily life, not just academic mathematicians. Yet, globally speaking, all people do not have regular access to or do not attend school. Ethnomathematics as a program of study offers one possibility – allowing researchers to examine what and how we teach mathematics in context of the school, culture and society.

Introduction

Modern 21st century mathematics represents the grammar and language of a particular culture that originated when people began working with quantity, measure, and shape in the Mediterranean regions of the world. Like any culture, mathematics comes with a unique grammar and way of thinking and seeing the world, and this mathematics is now coming into contact with other ways of thinking and interpreting our increasingly interdependent world. However, the conquest of the planet by modern mathematics is not without its consequences. A subtle sense of entitlement and a scientific cultural hegemony has spread faster than it sought to understand or come to terms with the thousands of traditions and time honored forms of thinking, calculating, and solving problems. We are all attracted to mathematics for its fine scientific, cultural, and even artistic qualities, and 21st century mathematics is allowing impressive marvels and scientific achievements. Yet at the same time, it has also enabled some the most horrific scientific and cultural disasters in the history of humanity.

The ethnomathematics community is now in position to move to the next step; that is to “walk the mystical way with practical feet”. In this paper we discuss concepts of multicultural education and multicultural mathematics that we have found useful in the interpretation of an ethnomathematics program for teacher professional development in our ongoing work in Brazil

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and the United States. We begin with an examination of multicultural education and ethnomathematics; and then we will outline both favorable and unfavorable arguments related to this paradigm.

Multicultural Education

Diverse concepts have emerged over the last few decades that have described the diverse programs and practices related to multiculturalism. We find the following definition useful. Multicultural education is…

> Consequently, we may define multicultural education as a field of study designed to increase educational equity for all students that incorporates, for this purpose, content, concepts, principles, theories, and paradigms from history, the social and behavioral sciences, and particularly from ethnic studies and women studies (Banks & Banks, 1995, p. xii).

Multicultural mathematics is the application of mathematical ideas to problems that confronted people in the past and that are encountered in present contemporary culture. In attempting to create and integrate, multicultural mathematics materials related to different cultures and draw on students own experiences into the regular instructional mathematics curriculum, WG-10 on Multicultural/Multilingual classrooms at ICME-7, in 1992 pointed out aspects of multicultural mathematics:

> An aspect of multicultural mathematics is the historical development of mathematics in different cultures (e.g. the Mayan numeration system). Another aspect could be prominent people in different cultures that use mathematics (e.g. an African-American biologist, an Asian-American athlete). Mathematical applications can be made in cultural contexts (e.g. using fractions in food recipes from different cultures). Social issues can be addressed via mathematics applications (e.g. use statistics to analyze demographic data (p.3-4).

The growing body of literature on multicultural education is stimulated by concerns for equity, equality, and excellence as part of a context of diversity. Teachers realize that students become motivated when they are involved in their own learning. This is especially true when dealing directly with issues of greatest concern to themselves (Freire, 1970). The challenge that many western societies face today is to determine how to shape a modernized, national culture that has integrated selected aspects of traditional cultures that coexist in an often delicate balance. This increased cultural, ethnic, and racial diversity provides both an opportunity and challenge to societies and institutions, with questions related to schooling forming an integral part of this question.

Ethnomathematics a Program

The inclusion of mathematical ideas from different cultures around the world, the acknowledgment of contributions that individuals from diverse cultures have made to mathematical understanding, the recognition and identification of diverse practices of a mathematical nature in varied cultural procedural contexts, and the link between academic mathematics and student experiences should become a central aspect to a complete study of mathematics. This is one of the most important objectives of an ethnomathematics perspective in mathematics curriculum development. Within this context, D'Ambrosio has defined ethnomathematics as,

> The prefix *ethno* is today accepted as a very broad term that refers to the social-cultural context, and therefore includes language, jargon, and codes of behavior, myths, and symbols. The derivation of *mathema* is difficult, but tends to mean to explain, to know, to understand, and to do activities such as ciphering, measuring, classifying, ordering, inferring, and modeling. The suffix *tics* is derived from *techne*, and has the same root as art and technique" (D'Ambrosio, 1990, p.81).

In this case, *ethno* refers to groups that are identified by cultural traditions, codes, symbols, myths and specific ways to reason and to infer and *mathematics* is more than counting, measuring, classifying, inferring or modeling. Ethnomathematics forms the intersection set between cultural anthropology and institutional mathematics and utilizes mathematical modeling to solve real-world problems.

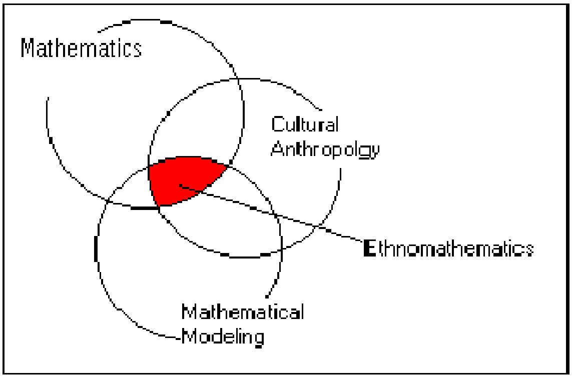

Figure 1: Ethnomathematics as an intersection of three disciplines

Essentially a critical analysis of the generation and production of mathematical knowledge and the intellectual processes of this production, the social mechanisms of institutionalization of knowledge (academics), and its transmission (education) are essential aspects of the program.

General Arguments

Multicultural education presents itself as a contemporary pedagogical trend in education. This approach allows a number of educators to deplore multicultural education and express numerous fears that this trend may represent a pulling away from certain cultural norms, even though some social realities underlie the need for many multicultural efforts to reform curricula.

The world's economy is becoming increasingly globalized; yet, traditional curricula neglects contributions made by the world's non-dominant cultures. Given these conditions, a multicultural approach may be seen as giving new, expanded, and often complicated definitions of a society's unique experiences. Multicultural education can reshape our greater cultural identity in a positive way (Banks, 1999; D'Ambrosio, 1995; Zaslavsky, 1996) by requiring the inclusion of a diversity of ethnic, racial, gender, social classes, as well as the practices and problems of the student's own community (D'Ambrosio, 1998; Zaslavsky, 1996). It helps students to understand the universality of mathematics, while revealing mathematical practices of day-to-day life, preliterate cultures, professional practitioners, workers, and academic or school mathematics. It can do this by taking into account historical evolution, and the recognition of the natural, social and cultural factors that shape human development (D'Ambrosio, 1995).

A Summary of Favorable Arguments

Education

Multicultural education promotes the rights of all people, no matter their sexual orientation, gender, ethnicity, race, and socio-economic status. It does this in order to allow learners to enable students to understand issues and problems of our diverse society (D'Ambrosio, 1990, 1995; Croom, 1997; Fasheh 1982; Zaslavsky, 1991, 1996, 1998). Through increasingly sophisticated multicultural experiences, students learn to make contributions and learn to appreciate the achievements of other cultures (D'Ambrosio, 1990, Joseph, 1991, Zaslavsky, 1996).

Mathematics

Multicultural mathematics education / ethnomathematics deals with both content and the process of curriculum, classroom management, teacher expectations, professional development, and relationships among teachers, administrators, students, and the community (Borba, 1990; D'Ambrosio, 1985, 1990, 1995, 1998; Zaslavsky, 1998). This approach allows students to make connections with historical developments of mathematics and the contributions made by diverse groups and individuals.

Students

Students need to be encouraged to develop skills in critical thinking and analysis that can be applied to all areas of life. These skills include vital issues involving health, environment, race, gender, and socioeconomic class (D'Ambrosio, 1990, 1998; Freire, 1970; Zaslavsky, 1998). Bassanezi (1990, 1994), Borba (1990), D'Ambrosio (1990, 1998), and Zaslavsky (1998) agree that ongoing contact of students with diverse ways of thinking and doing mathematics, will raise interest in learning required content, by having students apply mathematical concepts to future professional contexts and by facilitating student performance (Bassanezi, 1990, 1994; NCTM, 1989; Zaslavsky, 1990). In an ethnomathematics program students develop abilities, increased creativity, and a sound set of research habits. They will be able to develop a capacity to create a hypothesis (Bassanezi, 1990, 1994; D'Ambrosio, 1990, 1993; Biembengut, 1999; Hogson, 1995). Multicultural mathematics contributes to the development of student capacity by selecting data and subsequent adaptation to their needs (Biembengut, 1999, Croom, 1997; Hodgson, 1995), by encouraging contact with biology, chemistry, physics, geography, history, and language (Bassanezi, 1990, 1994; D'Ambrosio, 1995; Zaslavsky, 1991, 1993, 1996, 1998), and by developing work in groups, sharing tasks, learning how to take-in criticism and alternate opinions, respecting the decisions of others and the group, and by facilitating student interactions in a globalized society. Students share global and interactive visions necessary to develop successful mathematical content (D'Ambrosio, 1993; Bassanezi. 1990, 1994; Freire, 1970, Gerdes, 1988, 1988a; Zaslavsky, 1998).

Teachers and Educators

Multiculturalism encourages further intellectual development of the teacher. It also encourages long-term learning through a diversity of experiences. Teachers are characterized as facilitators / advisers of the mathematics learning process (Bassanezi, 1990, 1994; Biembengut, 1999; Hogson, 1995). Biembengut (1999) and Hogson (1995) stated that teachers and students discovered a process of understanding mathematics together. This context allows students to learn mathematics content through varied experiences related to the cultural, historical, and scientific evolution of mathematics.

A Summary of Unfavorable Arguments

Education

For several years it has been argued that traditional uses of the school curriculum do not foster genuine dispositions for realistic mathematics in students (Davis, 1989). Yet the same argument is used against attempts to make ethnomathematics useful to educators. Other concerns are related to the lack of enough time to develop content that enables teachers to execute pre-established pedagogical plans. It is difficult to mix multicultural education, ethnomathematics, benchmarks, standards and goals related to standardized testing that are based in traditional school mathematics (Burak, 1994; Pedroso, 1998). Concerns related to the application of ethnomathematics as pedagogical action include:

- Few, if any, textbooks and other materials about multicultural mathematics are in use in classrooms.
- A scarcity of university multicultural mathematics and ethnomathematics courses leave teachers and researchers unprepared to argue this issue.
- Few, if any, assessment instruments are appropriate to this new curriculum model.

- There is a danger of ethnomathematics being taught as folkloristic introductions to real mathematics.
- There exists great confusion between what is multicultural and ethnomathematics
- Much of the curricula represent a shallow, superficial learning with a sense of "multicultural" based upon "exposure to diversity".

Students

Many students have difficulty in group or cooperative learning. Many learners are unsure about how to work without the *traditional* classroom structure, and have difficulty assimilating several subjects simultaneously. Because of the traditional passive aspect of schooling, students often do not have the habit of formulating questions (Burak, 1994; Pedroso, 1998).

Teachers

Often because of the lack of their own personal experience, many educators are reticent to try cross-cultural methods, and their academic training in mathematics. Many educators are timid, and are reticent to attempt de-emphasizing traditional authority in favor of group work (Burak, 1994; Cross & Moscardini, 1985; Pedroso, 1998). Other reasons relate to issues of time for planning lessons (Burak, 1994; Pedroso, 1998; Zaslavsky, 1998). Many educators are not familiar with the interface between mathematics and other subjects, and certainly the reverse is equally true. Many educators are not prepared to employ practices that will enable underserved and underrepresented groups to learn mathematics (Burak, 1994, Zaslavsky, 1994).

Summary

The inclusion of multicultural mathematics and ethnomathematics continues according to the history of research in this area simply because the growing migration, immigration and diversity of our populations demand it. However, it can be negative when it restricts ethnic groups to stereotypes and leaves us unprepared to participate in academic endeavors. It is equally negative when it waters down mathematics content in general. Multicultural education seeks to recognize the contributions, values, rights, and the equality of opportunities of all groups that compose a given society. Educators can begin to develop the ideal equality among students and build a foundation for promoting academic excellence for all students (Croom, 1997). In an earlier work, Orey stated,

> A multicultural perspective on mathematical instruction should not become another isolated topic to add to the present curriculum content base. It should be a philosophical perspective that serves as both filter and magnifier. This filter/magnifier should ensure that all students, be they from minority or majority contexts will receive the best mathematics background possible. (Orey, 1989, p.7).

We recommend that the above questions continue to be debated in order to develop inclusive paths of further development of mathematics, our societies and the schools therein. As well, we hope that the Working group may come to some consensus in regards to the confusion between what is multicultural and ethnomathematics. Sociological questions about the relationships between institutionally dominant majority and minority cultures need be reflected by our increasingly globalized world.

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MATHEMATICAL COGNITION IN AND OUT OF SCHOOL FOR ROMANY STUDENTS

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First hand material that is collected on the spot, in a multicultural community in Athens, is used to demonstrate the relation between the mathematical cognition acquired by Romany within their community and mathematics learning of Romany students in school context. The fact that the formal education contemn or ignore the special cognition with which Romany students come to school is connected not only with their low school aptitude but also with the preservation of their marginal role in school as well as in the society.

Introduction

Until recently, the notion that mathematics is a culture free cognition had a major consequence: the attribution of the minority students' - students with cultural diversity - low aptitude to individual characteristics or diminished effort. In the last decades, several approaches such as ethnomathematics have manifested that cognitive as well as culture factors come together forming a coherent whole.

In Greece - as in the majority of countries - a lot of minority groups experience exclusion of formal education, quit attendance, fail in school tests or they are of low aptitude. One of these groups with cultural peculiarities is Romany people. Although rhetorically, a speech about equality before the law is articulated, in practice Romany students and formal education are considered to be incompatible. This view expressed by the State as well as the contempt of the informal cognition Romany students acquire in their community's context, result in a great proportion of illiterate - as well as mathematically illiterate - Romany people.

The orientation of this presentation is to examine the connection between cultural context of Romany community and school mathematics. Empirical evidence for this study was drawn from an ethnographic research, which was conducted due to a Ph.D. dissertation: “The Connection Between Cultural

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Context and Teaching/Learning of Mathematics: An Ethnographic Study on a Class of Romani Students and on their Community of Origin".

The study group was Romany students and their community of origin from Zephyri, a marginal multicultural district of Athens, Greece.

Among the main issues examined are the following:

a) How Romany people's way of live affects the production and use of mathematics notions and procedures?
b) What about Romany students' informal cognition acquired in their everyday life?
c) Which is the compatibility and incompatibility between formal and informal cognition?

Theoretical points

In the last decades, research is interested in different forms of mathematical cognition acquired in several contexts. According Abreu (1993), "very little is known about the conditions in which children home knowledge is evoked and how it interacts with school knowledge. The inverse is also true, in the sense that the way school knowledge interacts with out-of-school knowledge is also a subject under investigation". Fernandes talking about school role regarding formal cognition and the everyday one notices: "School socializes, very early, students into knowledge frames, which discourage connections with every day realities".

D' Ambrosio (1985) speaks about the crucial role of ethnomathematics within curriculum, presenting students with the opportunity to combine and integrate the self-generated methods they use in 'real' situations with school taught algorithms. He emphasizes in the need to acknowledge students' coping mechanisms and to consider the way that the individuals manage situation in life.

Jo Boaler talking about the role of the context pinpoints the complicate role of context: "Contexts have the power to form a barrier or bridge to understanding and it is this realization which prompts consideration of the range and complexity of influences upon a student's transfer of mathematics. It is also this realization that should ensure that contexts are used in learning, in order to provide bridges and to help break down barriers. Students construct their own meaning in given situations and an awareness of this should accompany decisions about the nature and variety of contexts used and about the openness of tasks. For if students determine their own direction and resolution in an activity then they will be able to determine their own goals.

This formation of goals means that students bring their own 'context' to a task and this must, ultimately, have personal meaning for them. Importantly the use of contexts must be accompanied by a reflection that the context of a task is capable of transforming students' perception, goals and subsequent choice of mathematical procedure". (Boaler, 1993).

Romany students in classroom and their community of origin in every day life

What is going to be discussed here is the way Romany students understand and use mathematics in school context and how this is connected with their cultural context.

The class where the research was conducted was a first grade class of Romany children. In the school however, there were also mixed—cultural and linguistic—background classes[1]. In every day life children spoke their native tongue, Roma, whereas in class they had to speak exclusively Greek. The particularity[2] of this class was its composition of children whose age was between 7-12 years old. The examination of this class was done in an anthropological perspective during the mathematical lessons and lasted a whole school year. During the two or three first months, students' number varied between thirty and thirty-four students. But after Christmas holiday the number stabilized between eight and ten.

Among the main reasons Romany students stop schooling is their way of live. Their **semi-nomadic way of live** is a great obstacle to their consistency in attendance. Speaking about our class, a part of students stopped school to follow their family, which moved to work in rural occupation, as the olive gathering is. We must notice that their difficulties concerning school are connected not only with the traditional moving around, but also their view about formal education.

Romany students' formal education is a very complicated matter. On the one hand they have to move either because they choose it, or because they are not accepted from the habitants of the place they choose to take up residence at. On the other hand they perceive education in a way that differs from the

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[1] We could speak about a multicultural school. We use this term not to assign equality before the law among children who come from different cultural groups, but just to assign a cultural diversity.

[2] The existence of a class with this composition is a cultural peculiarity. In 'normal' classes children start school at the age of seven.

common one: for Romany formal education is not a priority. They consider their survivor through everyday situation the most important thing.

Even nowadays, a great part of them disputes the value of formal education. They feel proud to go on with their lives regardless of the formal education and to acquire the cognition they consider important through their interaction within the borders of the community. Over and above, very often they consider school as an obstacle, because a student can't offer so much to the family when participating school. Apart from that school cognition is sometimes contradictory to the one acquired in the community.

What Romany people consider important is the cognition that has manageability. Students and their parents when asked 'about their ambitions of schooling' gave the following typical answers:

- to be able to read tablets in order to find the correct direction,
- to be able to read the time of ships' departure when they are on business travels or
- to improve their ability to calculate when they deal with money.

Through money dealings and in general through their involvement in their family's business Romany children acquire a plentiful corpus of informal cognition in a horizontal way of teaching**. Their socio-economic organization based upon family** give children the opportunity to be taught by experienced members of the community without conceiving this process as teaching. In their business occupations Romany people use and show interest in cognition necessary for them.

After talking with several members of the community it became obvious that they had working skills acquired through concrete practices such as mental calculations. For example, for the costermonger father of one student it was easy to do mental calculations in order to find the cost of five kilos of oranges each costing 130 drachmas: "with 100 … five hundred. The thirty with five (he means to multiply 5 to 30) one and half hundred … six and fifty hundred all of them. On the contrary their cognition about mathematical ideas or practices indifferent to them, their ability was of low working. The following discussion with a Romany man who was also a costermonger, is characteristic:

- *Do you know how many grammars are one- kilogram?*
- *100, no 1000. We don’t sell in grammars.*
- *Do you know how many grammars is the 1/4 of one-kilogram?*
- *This answer doesn't serve my anywhere. If it could serve me, I would know this answer.*

But in cases where they had to solve a problem in a concrete context they could invent suitable strategies. Another Romany person sold sacks with dung for flowers. Usually he sold 4 sacks for one thousand drachmas, or two for five hundred or one for two hundred and a half. Once, a client asked for thirty-four sacks. Firstly he felt awkward wondering how to calculate that. But then he invented a strategy of correspondence the "one to many": namely, he corresponded one thousand for every quadruplet. In this way he found the total cost.

Also students, as they acquire cognition in context that provide them with production of sense, they use common sense in problem solving not only in their every day life but also in school context.

The above strategy of corresponding "one to many", was a usual strategy for students too. A part of this class in the age of 10-12 years old had to solve the problem:

Basilis wanted to help his father to distribute[3] *apples in crates, which his father had got from the vegetable market. All the apples were 372 kg and every crate hold 20kg. How many crates does he need in order to put in all the apples?*

Three of them invented the same strategy as above. They used a correspondence of "one to many" and in that way figured out the correct number of crates. Concretely Apostolis, one of the students, was drawing lines on his desk: one for each crate (20 kilos).

R: *please, tell me Apostolis what are you doing here?*

A: *10 crates Miss.*

R: *How many kilos do the ten crates hold?*

A: *20 kg every crate.*

R: *So….*

A: *Well, 20, 40, …….180, 200.*

R: *And how many are there?*

A: *372*

He continued in the same way and found the correct number of crates. The forth student, Christos, invented a more complex solution. Namely, he used continuant subtraction. He got to subtract, from 372, 20 kilos at a time and so he found the number of the crates needed. Such a strategy premises a deep comprehension of the operation of division, managing to conceive it as continuant subtraction.

[3] We must notice that students weren't taught the operation of division and much more the typical algorithm of division, that is a procedure taught in the third grade.

The importance of this ability of Romany students became more obvious when we tested this problem to typical classes of fourth and fifth grade.

Regarding the fourth grade none of the students managed to find a correct solution using the correct algorithm. Some of them selected subtraction: 372 - 20 = 352, or addition: 372 + 20 = 392. These students although they have found results like that, they didn't wonder about the viability of their solution. They had just made the right application, without taking into consideration the essence of the problem. Even more interesting was the strategy of a girl, who used multiplication. She multiplied 20 x 372, finding as a result the number: 7.440. The girl tested this result doing the division: 7440: 20. Finding as quotient 372, she became sure about her correct solution.

Similar were the results of the fifth grade. Concretely, only the one third of the class managed to solve the problem correctly. The rest of the class either didn't select the correct operation or didn't apply correctly the algorithm. For example, results as 1 or 180 crates seemed to them as correct.

The ability of Romany students in using common sense as they acquire mathematics cognition in real life situation becomes more obvious in the next activity.

Students after having been taught the typical algorithm of addition and subtraction were called to solve the following problem: 24 - 18 = ? .

This was also written in this way:

  24
-18
___

to help them use the typical algorithm of subtraction.

All of them solved the problem without writing anything down, except of the result: 6. When being asked about their way of working they gave

Ch: we have *24, we take off 4 we have now 20. We take off the10*
following answeres: *and we have 10, we take off 4 more and 6 remain*
Typical representation: [(24 - 4) - 10] – 4 = 6
S: *we put 2 more to 18 and they become 20, and then I have other 4 and they become 24.*
Typical representation: 18 + 6 = 18 + 2 + 4 = (18 + 2) + 4 = 24
G: look, we have *18 and 24. I take off 10 from 18 and also ten from 24. The rest of 18 are 8. From 8 to ten is 2, and 4 (he means from 14) they become 6.*

Typical representation: $24 - 18 = (24 – 10) - (18 –10) = 14 – 8 = (10 + 4) – (10 – 2) = 4 + 2 = 6$.

The way Romany students negotiated the above activity shows us that for them it is easy to use mental calculations to solve problems not only in real life context but also in classroom. The fact that they acquire mathematical cognition in context that makes sense to them appears to make them able to transfer this cognition in other context as well, using common sense.

Their ability in mental calculation is also connected with **the orality that characterizes their culture**. The fact that they don't have a written language comprises one of the main cultural peculiarities that affect the way of school learning in a negative as well as a positive way. This peculiarity operates in a positive way since not only does it make students memorize a lot of information such as a shopping list but it also helps them to use mental calculations.

Concluding points

The orality of language, the semi-nomadic way of life, the socio- economic organization as well as the way the perception of formal education are the main cultural peculiarities that inform Romany identity. These cultural peculiarities affect Romany children mathematics education both in every day life and within the classroom.

Regarding every day life context Romany children acquire a lot of informal cognition though their involvement in family business. This informal cognition that they bring in the school context, as well as their difficulty in using formal ways for negotiating mathematical notions and procedures causes Romany students' diversity in school.

As we have already seen Romany students are more efficacious in comparison to non-Romany students in problem solving, if they are permitted to use their own algorithms. Romany students through their experience focus on the semasiology of the problem and not in the syntactic - something that happens regarding the non-Romany students. On the contrary, they face difficulties in using typical algorithms. It isn't easy for them to use formal ways in doing mathematics mostly because they aren’t familiar with the written symbols as well as they manage to solve problems using strategies occupied in context and they are usually effective.

The fact Romany students carry different informal cognition in school and also their different ways of learning are elements ignored from the formal education and it has as a consequence their school failure. An ethnomathematics approach, which could embody ethnomathematical ideas in curriculum and textbooks as well as a suitable education for the educators so that they could identify students cultural diversity - and their way of learning - could empower Romany - and general students that come form minority and marginal groups - in school and in greater society.

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