

# Preface

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This booklet collects the notes of the course I gave in the graduate school of Mathematics of Pisa during the winter 2000-2001.

My aims for this course were both to describe the basic theory of discrete hyperbolic dynamical systems, and, at the same time, to present several examples — several concrete instances of the general theory — helping the students to get a good intuitive grasp on the subject.

For this reason the first two chapters are devoted both to introduce the main concepts (contractions, topological transitivity, chaos, structural stability, recurrence and so on), and to work out in detail several examples: linear maps, toral automorphisms, expanding maps of the circle, Smale’s horseshoe, and others. We really start the general theory in the third chapter, devoted to the proof of the Hadamard-Perron theorem for not necessarily invertible maps; from this theorem we derive stable and unstable manifolds theorems, both for hyperbolic fixed points and for general hyperbolic sets. In the fourth chapter we finally define what we mean by “hyperbolic dynamical system” (even though we already used the word “hyperbolic” before), and we prove several important results: Anosov’s closing lemma, the spectral decomposition theorem, the strong structural stability of hyperbolic sets, and others. The fifth chapter has a somewhat different nature: it collects talks given (and written) by the students as final examination, presenting a few scattered results (the Hartman-Grobman theorem, the specification theorem, etc.). Here and there I also put in the text some exercises; most of them are routine, some less so.

As often happens for lecture notes, the presentation is not as polished as it could have been. In particular, I did not even try to mention the original discoverers of the results presented here, or to quote the original papers. The choice and the order of the arguments owe a lot to the wonderful book by A. Katok and B. Hasselblatt, **Introduction to the modern theory of dynamical systems**, Cambridge University Press, Cambridge, 1995. I also found very useful M. Shub’s **Global stability of dynamical systems**, Springer, Berlin, 1987.

It is a pleasure to thank both Fabrizio Broglia for giving me the possibility to teach this course, and all the students involved — in particular Claudio Bonanno, Michele Cerulli, Mirko Maracci e Roberto Stasi who followed my ramblings up to the end. I heartily thanks Carlo Carminati and Filippo Bracci for carefully reading the notes, pointing out many mistakes (I was only testing their attention, of course. . .). And, last but not least, I am really grateful to Francesca Tovenà for the right suggestion at the right moment, and to Adele Manzella for putting up with me through all this.

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*Marco Abate*