Algebra Lineare - Esercizi del 13/11/08

Determinare $[f]_{\mathcal{B}}^{\mathcal{C}}$:

(1) $f: \mathbb{R}^3 \to \mathbb{R}^2$ $\quad \mathcal{B} = \left(\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} -4 \\ 3 \\ -2 \end{pmatrix}, \begin{pmatrix} 5 \\ -1 \\ -3 \end{pmatrix} \right)$ $\quad \mathcal{C} = \left(\begin{pmatrix} 4 \\ 3 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \end{pmatrix} \right)$

$f(x) = \begin{pmatrix} 2x_1 - 3x_2 + 4x_3 \\ 5x_1 + x_2 - 2x_3 \end{pmatrix}$

(2) $f: \mathbb{R}^2 \to \mathbb{R}^3$ $\quad \mathcal{B} = \left(\begin{pmatrix} 4 \\ 11 \end{pmatrix}, \begin{pmatrix} -2 \\ 7 \end{pmatrix} \right)$ $\quad \mathcal{C} = \left(\begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}, \begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ -5 \end{pmatrix} \right)$

$f(x) = \begin{pmatrix} 3x_1 - 2x_2 \\ x_1 + 3x_2 \\ -2x_1 + 4x_2 \end{pmatrix}$

(3) $f: \mathcal{S}_2(\mathbb{R}) \to \mathbb{R}^2$ $\quad \mathcal{B} = \left(\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 3 \\ 3 & 4 \end{pmatrix} \right)$

$\mathcal{C} = \left(\begin{pmatrix} -3 \\ 7 \end{pmatrix}, \begin{pmatrix} 2 \\ -5 \end{pmatrix} \right)$ $\quad f(A) = \begin{pmatrix} a_{11} + 2a_{12} + 3a_{22} \\ -3a_{11} + 4a_{21} - 5a_{22} \end{pmatrix}$

(4) $f: \mathbb{R}^2 \to \mathcal{A}_3(\mathbb{R})$ $\quad \mathcal{B} = \left(\begin{pmatrix} 5 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ -7 \end{pmatrix} \right)$

$\mathcal{C} = \left(\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & 0 \\ -2 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 3 & 1 \\ -3 & 0 & -1 \\ -1 & 1 & 0 \end{pmatrix} \right)$

$f(x) = \begin{pmatrix} 0 & x_1 - x_2 & 3x_1 - x_2 \\ x_2 - x_1 & 0 & x_2 - 2x_1 \\ x_2 - 3x_1 & 2x_1 - x_2 & 0 \end{pmatrix}$

(5) $f: \mathbb{R}^2 \to \{x \in \mathbb{R}^3 : x_1 + x_2 + x_3 = 0\}$

$\mathcal{B} = \left(\begin{pmatrix} 2 \\ 7 \end{pmatrix}, \begin{pmatrix} 3 \\ -1 \end{pmatrix} \right)$ $\quad \mathcal{C} = \left(\begin{pmatrix} 2 \\ -5 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 4 \\ -5 \end{pmatrix} \right)$

$$f(y) = \begin{pmatrix} y_1 + 2y_2 \\ -3y_1 + 5y_2 \\ 2y_1 - 7y_2 \end{pmatrix}$$

(6) $f : \{ x \in \mathbb{R}^3 : x_1 + 2x_2 - x_3 = 0 \} \to \mathbb{R}^3$

$$\mathcal{B} = \left(\begin{pmatrix} 4 \\ 1 \\ 6 \end{pmatrix}, \begin{pmatrix} 5 \\ -2 \\ 1 \end{pmatrix} \right) \qquad \mathcal{C} = \left(\begin{pmatrix} 3 \\ 1 \\ -4 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}, \begin{pmatrix} 1 \\ -5 \\ 2 \end{pmatrix} \right)$$

$$f(x) = \begin{pmatrix} x_1 + 2x_2 - 4x_3 \\ 2x_1 - x_2 + x_3 \\ -4x_1 + 2x_3 \end{pmatrix}$$

(7) $f : \mathbb{R}_{\leq 2}[t] \to \mathbb{R}^2$ $\qquad \mathcal{B} = (1+t,\ 1+t^2,\ t+t^2)$

$$\mathcal{C} = \left(\begin{pmatrix} 5 \\ 11 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right) \qquad f(p(t)) = \begin{pmatrix} p(1) - p'(2) \\ p''(\sqrt[4]{5}) + 2p(-1) \end{pmatrix}$$

(8) $f : \{ x \in \mathbb{R}^3 : 2x_1 + 3x_2 - 4x_4 = 0 \} \to \mathbb{R}_{\leq 2}[t]$

$$\mathcal{B} = \left(\begin{pmatrix} -3 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} \right) \qquad \mathcal{C} = (1 + t - t^2,\ 1 - 2t + 3t^2,\ 3 + t + t^2)$$

$$f(x) = (x_1 - 2x_3) + (2x_2 + x_3)t + (x_1 + x_2 + x_3)t^2$$

(9) $f : \{ A \in M_{2\times 2}(\mathbb{R}) : tr(A) = 0 \} \to \{ p(t) \in \mathbb{R}_{\leq 2}[t] : p'(-1) = 0 \}$

$$\mathcal{B} = \left(\begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix}, \begin{pmatrix} 0 & 4 \\ -7 & 0 \end{pmatrix}, \begin{pmatrix} 5 & 1 \\ -3 & -5 \end{pmatrix} \right)$$

$\mathcal{C} = \left(1+2t+t^2 ,\ 3-t-\frac{1}{2}t^2 \right)$

$f(A) = (a_{11} + 2a_{12} - a_{21}) + (a_{11} + 2a_{21} - a_{22})\,t$

$\qquad\qquad + (a_{11} + a_{21})\,t^2$

Determinare le matrici di cambiamento da $\mathcal{B}$ a $\mathcal{B}'$ e da $\mathcal{B}'$ a $\mathcal{B}$ verificando che sono l'inversa l'una dell'altra:

(10) $V = \mathbb{R}^2 \quad \mathcal{B} = \left(\begin{pmatrix} 4 \\ 1 \end{pmatrix} \begin{pmatrix} 3 \\ -7 \end{pmatrix} \right), \quad \mathcal{B}' = \left(\begin{pmatrix} -2 \\ 5 \end{pmatrix}, \begin{pmatrix} 3 \\ 4 \end{pmatrix} \right)$

(11) $V = \mathbb{R}^3 \quad \mathcal{B} = \left(\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 3 \\ -2 \end{pmatrix} \right), \quad \mathcal{B}' = \left(\begin{pmatrix} 5 \\ 2 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ 4 \\ -2 \end{pmatrix}, \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix} \right)$

(12) $V = \{ x \in \mathbb{R}^3 : 2x_1 + 3x_2 - 5x_3 = 0 \}$

$\mathcal{B} = \left(\begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 5 \\ 3 \end{pmatrix} \right), \quad \mathcal{B}' = \left(\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix} 7 \\ 3 \\ -1 \end{pmatrix} \right)$

(13) $V = \mathcal{S}_2(\mathbb{R}) \quad \mathcal{B} = \left(\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -2 \\ -2 & 1 \end{pmatrix}, \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix} \right)$

$\mathcal{B}' = \left(\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 2 \\ 2 & -1 \end{pmatrix}, \begin{pmatrix} 3 & -2 \\ -2 & 2 \end{pmatrix} \right)$

(14) $V = \{ p(t) \in \mathbb{R}_{\leq 2}[t] : p(-1) = 0 \}$

$\mathcal{B} = \left(1+t,\ 1-t^2 \right) \quad \mathcal{B}' = \left(1+2t+t^2,\ 2-t-3t^2 \right)$