Algebra Lineare - Esercizi del 30/10/08

(1) Completare a una base di V i seguenti vettori $v_1, v_2, \ldots$

(a) $V = \mathbb{R}^4$, $v_1 = \begin{pmatrix} 1 \\ 2 \\ -3 \\ 5 \end{pmatrix}$, $v_2 = \begin{pmatrix} 1 \\ 0 \\ 4 \\ -8 \end{pmatrix}$

(b) $V = \mathbb{R}^5$, $v_1 = \begin{pmatrix} -1 \\ 3 \\ 2 \\ 4 \\ 1 \end{pmatrix}$, $v_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 3 \\ 2 \end{pmatrix}$, $v_3 = \begin{pmatrix} -8 \\ 9 \\ 1 \\ -1 \\ -4 \end{pmatrix}$

(c) $V = \{x \in \mathbb{R}^4 : 2x_1 - 3x_2 + 4x_3 - 5x_4 = 0\}$ $v_1 = \begin{pmatrix} 6 \\ 2 \\ 1 \\ 2 \end{pmatrix}$

(d) $V = \left\{x \in \mathbb{R}^4 : \begin{array}{l} 3x_1 - x_2 + 2x_3 + 3x_4 = 0 \\ 2x_1 + 4x_2 - x_3 + 2x_4 = 0 \end{array}\right\}$ $v_1 = \begin{pmatrix} -3 \\ 1 \\ 2 \\ 2 \end{pmatrix}$

(e) $V = \left\{x \in \mathbb{R}^4 : \begin{array}{l} 2x_1 + 4x_2 - 5x_3 + x_4 = 0 \\ x_1 - 2x_2 + 7x_3 + 4x_4 = 0 \end{array}\right\}$ $v_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ -1 \end{pmatrix}$

(f) $V = \mathcal{S}_2(\mathbb{R})$, $v_1 = \begin{pmatrix} 1 & 2 \\ 2 & 0 \end{pmatrix}$ $v_2 = \begin{pmatrix} 0 & 3 \\ 3 & -1 \end{pmatrix}$

(g) $V = \mathcal{F}(\{a,b,c,d\}, \mathbb{R})$ $v_1 = \begin{cases} a \longmapsto 2 \\ b \longmapsto -3 \\ c \longmapsto 0 \\ d \longmapsto 7 \end{cases}$ $v_2 = \begin{cases} a \longmapsto 1 \\ b \longmapsto 2 \\ c \longmapsto 5 \\ d \longmapsto -3 \end{cases}$

(h) $V = \mathbb{R}[t]$ $v_1 = 1 - t^2$, $v_2 = t - 7t^3$, $v_3 = 2 - 4t + t^3$

(i) $V = \{p(t) \in \mathbb{R}[t] : p^{(v)}(\cancel{t}) = 0,\ p(1) = 0\}$

$v_1 = 1 - t + 3t^2 - 2t^3 - t^4$

(2) Estrarre dai generatori $v_1, v_2, \dots$ una base di V

(a) $V = \mathbb{R}^3$ $\quad v_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, v_2 = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}, v_3 = \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix}$

$v_4 = \begin{pmatrix} -1 \\ 5 \\ -10 \end{pmatrix}, v_5 = \begin{pmatrix} +5 \\ -4 \\ +11 \end{pmatrix}, v_6 = \begin{pmatrix} -1 \\ 4 \\ 7 \end{pmatrix}$

$v_7 = \begin{pmatrix} \pi \\ \sqrt{3} \\ 11 \end{pmatrix}$

(b) $V = \{x \in \mathbb{R}^4 : x_1 + x_2 = x_3 + x_4\}$

$\begin{pmatrix} 2 \\ -3 \\ -4 \\ 3 \end{pmatrix} \begin{pmatrix} 1 \\ 7 \\ -4 \\ 12 \end{pmatrix} \begin{pmatrix} 5 \\ 18 \\ -16 \\ 39 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} -1 \\ -8 \\ 10 \\ -19 \end{pmatrix}$

(c) $V = \{p(t) \in \mathbb{R}[t] : \deg(p(t)) \leq 3\}$

$-t, \; 1-t+t^2, \; 5+2t-t^3, \; 6+t^2-t^3,$

$-8t+5t^2+t^3, \; 2-t+3t^2-4t^3$

(d) $V = \mathcal{Q}_3(\mathbb{R})$

$\begin{pmatrix} 0 & 1 & -3 \\ -1 & 0 & -2 \\ 3 & 2 & 0 \end{pmatrix} \begin{pmatrix} 0 & -7 & 4 \\ 7 & 0 & -1 \\ -4 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 9 & -10 \\ 9 & 0 & -5 \\ -10 & -5 & 0 \end{pmatrix} \begin{pmatrix} 0 & -1 & 3 \\ 1 & 0 & -2 \\ -3 & +2 & 0 \end{pmatrix} \begin{pmatrix} 0 & -7 & -11 \\ 7 & 0 & 3 \\ 11 & -3 & 0 \end{pmatrix}$

(e) $V = \{x \in \mathbb{R}^3 : x_1 - 2x_2 + 3x_3 = 0\}$

$\begin{pmatrix} -1 \\ 4 \\ 3 \end{pmatrix} \begin{pmatrix} 5 \\ 2 \\ -1 \end{pmatrix} \begin{pmatrix} 7 \\ -6 \\ -7 \end{pmatrix} \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$

(f) $V = \{x \in \mathbb{R}^4 : x_1 + x_3 = x_2 + x_4\}$

$\begin{pmatrix} 2 \\ -3 \\ -1 \\ 4 \end{pmatrix}, \begin{pmatrix} -1 \\ 5 \\ 8 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ 7 \\ 15 \\ 8 \end{pmatrix}$