

"Geometria" - Esercizi del 19/05/10

(1) Dire se le seguenti matrici siano normali, e in tal caso se siano unitarie (discutendo al variare di  $k \in \mathbb{C}$  quando presente):

$$(a) \begin{pmatrix} 2+i & 1 \\ 3i & 1-i \end{pmatrix}$$

$$(b) \begin{pmatrix} 1-\frac{i}{2} & -1+\frac{i}{2} \\ \frac{i}{2}-1 & 1-\frac{i}{2} \end{pmatrix}$$

$$(c) \frac{1}{10} \begin{pmatrix} 7-7i & 1+i \\ 1+i & 7-7i \end{pmatrix}$$

$$(d) \frac{1}{2\sqrt{2}} \begin{pmatrix} 1-i & -\sqrt{2}+\sqrt{2}i & i-3 \\ -\sqrt{2}+\sqrt{2}i & 2+2i & +\sqrt{2}-\sqrt{2}i \\ 3-i & \sqrt{2}+\sqrt{2}i & 1-i \end{pmatrix}$$

$$(e) \begin{pmatrix} \frac{1}{2} & -\frac{i}{\sqrt{2}} & -\frac{i}{2} \\ -\frac{i}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{i}{2} & -\frac{1}{\sqrt{2}} & \frac{1}{2} \end{pmatrix}$$

$$(f) \frac{1}{4} \begin{pmatrix} 2k^2+ik+1 & -\sqrt{2}i \cdot k + \sqrt{2} & -2i \cdot k^2 - k + i \\ -\sqrt{2}i \cdot k + \sqrt{2} & 2i \cdot k + 2 & -\sqrt{2}k + \sqrt{2}i \\ 2ik^2+k-i & -\sqrt{2}k - \sqrt{2}i & 2k^2+ik+1 \end{pmatrix}$$

(2) Stabilire se le seguenti matrici siano normali e in tal caso esibire una matrice unitaria che le diagonalizzi:

$$(a) \begin{pmatrix} 1-5i & -6i \\ 3i & 1+4i \end{pmatrix}$$

$$(b) \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$

$$(c) \frac{1}{10} \begin{pmatrix} 5+5i & 1+7i \\ -7-i & 5+5i \end{pmatrix}$$