

## **Notions of complexity for minimal subvarieties**

The recent, remarkable advances in the construction of minimal hypersurfaces in general ambient manifolds, either via min-max methods in the spirit of Almgren-Pitts as developed by Marques and Neves, or studying the interfaces arising as suitable singular limits of solutions to the Allen-Cahn equations as proposed by Guaraco, suggest an incredibly rich scenario and have inspired an impressive body of research directions, related to the problems of classifying minimal hypersurfaces under various sorts of ambient curvature conditions, studying their moduli spaces and the associated Weyl's laws, proving generic finiteness results and topological uniqueness theorems among others.

After a broad-spectrum introduction, I will outline a general project aimed at comparing different ways of quantifying the "complexity" of a minimal subvariety: I will present universal comparison results relating analytic data (like the Morse index, the value of the  $p$ -th eigenvalue of the Jacobi operator etc...), geometric data (the area, the Yamabe invariant etc...) and topological invariants (like e. g. the Betti numbers). We shall be concerned about the interactions between these different pieces of information and investigate whether these measures of complexity are equivalent in some suitable sense.

For instance, I will discuss the state of the art about Schoen's conjecture asserting that the Morse index of any (closed, embedded) minimal surface inside a 3-manifold of positive Ricci curvature should be bounded from below by an affine function of the genus (with universal coefficients) and, on the other hand, mention partial results for the conjecture by A. Ros classifying the possible topological types of index one minimal surfaces inside 3-manifolds of positive scalar curvature.

Analogous results have also been obtained for free boundary minimal hypersurfaces inside Riemannian domains.

This lecture is based on various results that have been obtained in collaboration with Lucas Ambrozio, Reto Buzano and Benjamin Sharp.