TRANSLATION SURFACES: FROM GEOMETRY TO SPECTRAL THEORY

MAURO ARTIGIANI AND PAOLO GIULIETTI

Introduction

Translation surfaces are a generalization of flat tori to higher genuses, with rich and interesting geometrical and dynamical properties. In fact, they can be seen both from a complex geometry point of view, stemming from the works of Teichmüller, Ahlfors and Bers, and also from a Euclidean geometry point of view, connecting to the works of the Russian school on low dimensional dynamics. These two different point of views have been fruitfully exploited in the last 50 years to obtain many deep and beautiful results.

In this course, we will introduce translation surfaces motivating their interest. Then, we will survey some of the classical results about them, focusing on the dynamical point of view: the geodesic flow on a translation surface is an important example of a *parabolic system*, in which nearby points diverge slowly from each other. We will stress the general philosophy of *renormalization*, which connects the study of the geodesic flow on a surface with the study of the geodesic flow on the moduli space of translation surfaces, which has a chaotic behavior that can be exploited to obtain many information on our initial flow.

Prerequisites are measure theory, some familiarity with complex analysis and functional analysis.

A sketch of the course is as follows:

(1) Introduction, motivation, brief survey of Teichmüller's theorems ([3, 7]);
(2) Basic definitions, examples, Masur's criterion, Sketch of Kerkhoff, Masur and Smillie ([4]);
(3) Veech's dichotomy, Examples of Veech surfaces (without proofs) ([5, 6]);
(4) Lyapunov exponents, motivations: Zorich's asymptotic flag. Forni's proof of Kontsevich-Zorich conjecture for genus 2 ([3, 7]);
(5) Detour, an explicit counterexample: the Eierlegende Wollmilchsau ([3]);
(6) Sketch of the exponential mixing of the Teichmüller flow (following Avila, Gouëzel and Yoccoz) ([1]);
(7) An introduction to Transfer operator Theory;
(8) Ruelle resonances for pseudo-Anosov transformations ([2]).

Practical Infos

The course will last 5 weeks, for a total of approximately 30 hours.

The first lecture will be on the 4th of June at 15,30 (Rome Time).

From the 7th June to the 10th of July the schedule is "Rome Time"

Date: May 28, 2021.

Monday 11,30 - 13,30
Wednesday 15,30 - 17,30
Friday 15,30 - 17,30

BEWARE: Due to the italian tradition of "quarto d'ora accademico" lectures begin 15 minutes after the scheduled time.

It will be held virtually on the official Teams platform, hosted by the university of Pisa. Some of the lectures will be streamed and recorded (for our own use) via the Zoom platform. Accordingly, the links will be provided both by mailing list and the Teams channel.

If you would like to attend the lectures, let us know by writing to paolo.giulietti@unipi.it or mauro.artigiani@urosario.edu.co

References

[1] Artur Avila, Sébastien Gouëzel, and Jean-Christophe Yoccoz. 'Exponential mixing for the Teichmüller flow'. *Publications Mathématiques. Institut de Hautes Études Scientifiques* 104 (2006), pp. 143–211. DOI: `10.1007/s10240-006-0001-5`.

[2] Frédéric Faure, Sébastien Gouëzel, and Erwan Lanneau. 'Ruelle spectrum of linear pseudo-Anosov maps'. *Journal de l'École polytechnique — Mathématiques* 6 (2019), pp. 811–877. DOI: `10.5802/jep.107`. URL: `https://jep.centre-mersenne.org/articles/10.5802/jep.107/`.

[3] Giovanni Forni and Carlos Matheus. 'Introduction to Teichmüller theory and its applications to dynamics of interval exchange transformations, flows on surfaces and billiards'. *Journal of Modern Dynamics* 8.3–4 (2014), pp. 271–436. DOI: `10.3934/jmd.2014.8.271`.

[4] Sébastien Gouëzel and Erwan Lanneau. 'Un théorème de Kerckhoff, Masur et Smillie : unique ergodicité sur les surfaces plates'. In: *École de théorie ergodique*. Ed. by Yves Lacroix, Pierre Liardet, and Jean-Paul Thouvenot. Paris: SMF, 2010, pp. 113–145. URL: `https://www.math.sciences.univ-nantes.fr/~gouezel/articles/kms.pdf`.

[5] Pascal Hubert and Thomas Schmidt. 'An introduction to Veech surfaces'. In: *Handbook of dynamical systems. Vol. 1B*. Ed. by Boris Hasselblatt and Anatole Katok. Amsterdam: Elsevier B.V., 2006, pp. 501–526.

[6] William A. Veech. 'Teichmüller curves in moduli space, Eisenstein series and an application to triangular billiards'. *Inventiones Mathematicae* 97.3 (1989), pp. 553–583. DOI: `10.1007/BF01388890`.

[7] Anton Zorich. 'Flat surfaces'. In: *Frontiers in number theory, physics, and geometry. I*. Ed. by Pierre Cartier et al. Berlin: Springer, 2006, pp. 437–583.

School of Engineering, Science and Technology, Universidad del Rosario, Carrera 6 #12C-16, Bogotá, Colombia
Email address: `mauro.artigiani@urosario.edu.co`

Dipartimento di Matematica, Università di Pisa, 56127 Pisa, Italia
Email address: `paolo.giulietti@unipi.it`