ON THE MORSE–SARD THEOREM FOR THE SHARP CASE OF SOBOLEV MAPPINGS AND APPLICATIONS IN FLUID MECHANICS

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The first part of talks is based on joint papers [1]–[3]. We establish Luzin N- and Morse–Sard properties for mappings $f\colon \mathbb{R}^n \to \mathbb{R}^m$ of the Sobolev–Lorentz class $\mathrm{W}^k_{p,1}$ with $k = n - m + 1$ and $p = \frac{n}{k}$ (this is the sharp case that guaranties the continuity of mappings; for values $k = n$, $p = 1$ the Sobolev–Lorentz class $\mathrm{W}^n_{1,1}(\mathbb{R}^n)$ coincides with the usual Sobolev space $\mathrm{W}^n_1(\mathbb{R}^n)$). Using these results we prove that almost all level sets of f are finite disjoint unions of C^1–smooth compact manifolds of dimension $n - m$ (despite of the fact that f itself is not C^1) .

These results helped in mathematical fluid mechanics — for the so-called Leray's problem, which remained open for more than 80 years (starting from the publication of the famous paper of Jean Leray 1933 [7]). Namely, for plane and axially symmetric spatial flows the existence theorem was proved for boundary value problem of stationary Navier-Stokes equations in bounded domains under necessary and sufficient condition of zero total flux (see [4]–[6]).

Recall that according to the mass conservation law the total flux (i.e. the amount of fluid flows through all the boundary components of the domain) should be zero, it is a necessary condition of solvability. However, J. Leray proved the existence of a solution under a stronger assumption that the flow of fluid through each component of the boundary is zero (this condition means the lack of sources and sinks). The case when the boundary value satisfies only the necessary condition of zero total flux (i.e. when the sources and sinks are allowed) was left open by him and the problem of existence (or nonexistence) of a solution for such case is known in the scientific community as *Leray's problem.*

References

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