Solvability for a class of semilinear elliptic Fuchsian PDEs

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We discuss solvability for semilinear elliptic equations of the form

$$Au = F(x, B_1u, \ldots, B_Ku) \text{ in } X \setminus \partial X, \quad Tu = g \text{ on } \partial X,$$

for a differential operator A that is Fuchsian with respect to ∂X, X being a C^∞ compact manifold with non-empty boundary ∂X, where A together with the boundary condition $Tu = 0$ is positive definite in the weighted L^2 space $H^{0,\delta}(X)$, for some $\delta \in \mathbb{R}$. The Fuchsian differential operators $B_1, \ldots, B_K$ are of orders strictly less than A, and the nonlinearity $F = F(x, \nu)$ is of at most polynomial growth in ν. Moreover, the linear surjective boundary map $T\colon D_+ \to \mathbb{R}^\mu$ factors through D_+/D_-, where D_+ and D_- are the maximal and minimal domains of A in $H^{0,\delta}(X)$, respectively; $\dim D_+/D_- = 2\mu$.

As solutions to the above problem are unbounded in general, the main step of our approach consists in an *a priori* description of the asymptotics of these solutions $u = u(x)$ as $x \to \partial X$.