Combinazione Lineare

Le matrici e i vettori si possono sommare, sse hanno le stesse dimensioni:

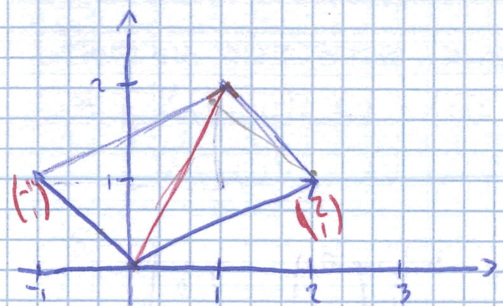
Formula: $Mat_{m \cdot k} + Mat_{m \cdot k} = Mat_{m \cdot k} \Rightarrow \underset{\substack{i \le m \\ j \le k}}{(a_{ij})} + \underset{\substack{i \le m \\ j \le k}}{(b_{ij})} = (c_{ij})$

$c_{ij} = a_{ij} + b_{ij}$

Oss: $k \cdot (a_{ij})_{ij} = (k a_{ij})_{ij}$

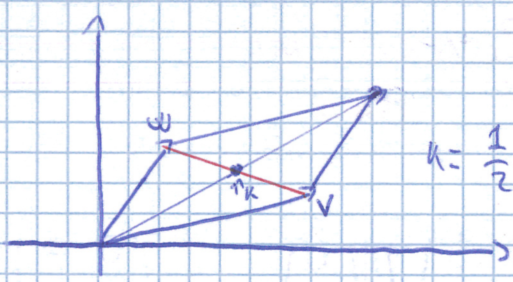
Geometricamente i vettori

I vettori si sommano con la regola del parallelogramma.



$$\begin{pmatrix} 2 \\ 1 \end{pmatrix} + \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

Esercizio: Dati 2 vettori,



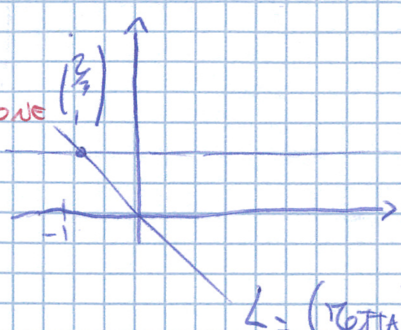
$k = \frac{1}{2}(v + w)$

Retta

2 modi di descrivere una retta nel piano passante per $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

1. $\begin{cases} ax + by = 0 \\ 3x + 2y = 0 \end{cases}$

Sottoforma di equazione



L = (retta)

② Metodo Parametrico

$$L = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \middle| \ 3x + 2y = 0 \right\} = \left\{ \begin{pmatrix} -\frac{2}{3}k \\ k \end{pmatrix} \middle| \ k \in \mathbb{R} \right\} = \left\{ k \begin{pmatrix} -\frac{2}{3} \\ 1 \end{pmatrix} \middle| \ k \in \mathbb{R} \right\}$$

Oss: Matrici A, B $\rightarrow$ $A \cdot B = B \cdot A$? no!

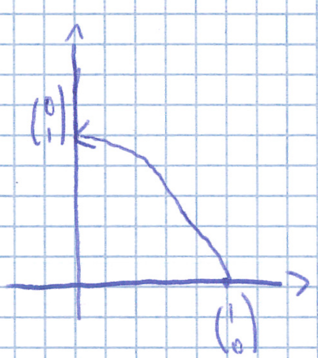
La proprietà commutativa non vale per le moltiplicazioni tra matrici.

Matrici come funzioni (lineari)

$$\underbrace{\begin{pmatrix} a & b \\ c & d \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} x \\ y \end{pmatrix}}_{V} = \underbrace{\begin{pmatrix} ax + by \\ cx + dy \end{pmatrix}}_{W} \Rightarrow AV = w$$

Quindi $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} a \\ c \end{pmatrix}$ $\qquad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} b \\ d \end{pmatrix}$

Rotazioni nel piano



$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} 0 & b \\ 1 & d \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} a & -1 \\ c & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

Quindi: $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$