

**A Chebyshev-based two-stage iterative method  
as an alternative to the direct solution of linear systems**

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We consider the solution of ill-conditioned symmetric and positive definite large sparse linear systems of equations. These arise, for instance, when using some symmetrizable preconditioning technique for solving a general (possibly unsymmetric) ill-conditioned linear system, or in domain decomposition of a numerically difficult elliptic problem.

We are also concerned with the consecutive solution of several linear systems with the same matrix and different right-hand sides. In such cases, the consecutive runs of some iterative methods like the conjugate gradient or the block conjugate gradient algorithms might be computationally prohibitive, and it might be preferable to use direct methods which are very well suited for this type of situation.

The purpose of our study is to analyse a two-phase approach: analogously to direct methods, it includes a preliminary phase which we call a ``partial spectral factorization phase'', followed by a ``cheap'' solution phase, both only based on numerical tools that are usually exploited in iterative methods. Therefore, we need not store the given ill-conditioned matrix explicitly but we need only use it to compute matrix-vector products. This is of particular interest in the case of very large sparse systems and permits an efficient implementation on distributed memory architectures.

Combining Chebyshev iterations with the block Lanczos algorithm, we propose a way to identify and extract precise information related to the ill-conditioned part of the given linear system, and to use it for the computation of extra solutions related to the same matrix with changing right-hand sides. The potential of this combination, which can be related to the factorization and direct solution of linear systems, is illustrated experimentally in the general case of the preconditioned iterative solution of unsymmetric sparse linear systems.