Recursive and/or iterative refinement for a superfast solver for real symmetric Toeplitz systems based on real trigonometric transformations

M.Van Barel, G.Codevico, G. Heinig

In this talk we present a superfast algorithm for the solution of a linear system of equations

$$T_n \mathbf{x} = \mathbf{b}$$

with a nonsingular real $n \times n$ symmetric Toeplitz coefficient matrix $T_n = [\, a_{|i-j|} \,]_{i,j=0}^{n-1}$. The solution is computed with computational complexity $O(n \log^2 n)$. Algorithms with this complexity can be found in the literature, in [10, 2, 3, 5, 1, 8, 14] and other papers (see [11]). The recent paper [12] describes an algorithm to solve positive definite Toeplitz systems with a slightly higher complexity $O(n \log^3 n)$. All these algorithms are based on the Fast Fourier Transform. In contrast to this, the new algorithm is based on real trigonometric transforms. Since the complexity for trigonometric transforms is essentially less than that one for complex FFT, even less than real FFT (see [13]), the new algorithm should be, if the implementation details are worked out properly, faster than previous ones.

The problem that turns out is the numerical in-stability of the algorithm connected to the Chebyshev basis and the Chebyshev nodes. To improve the stability we present two new techniques refining the intermediate solutions of the rational interpolation problem associated to the original matrix. The first one is based on the reinterpretation of the problem in terms of a tangential interpolation problem associated with a coupled-Vandermonde matrix. The second one is a sort of bootstrap method to refine the solution computed in each sub-tree of the divide-et-impera scheme.

The approach in this talk is based on the results from [7] and [4].

References

[1] G.S. Ammar, W.B. Gragg, Superfast solution of real positive definite Toeplitz systems, *SIAM J. Matrix Analysis Appl.* 9: 61-76 (1988).

[2] R.R. Bitmead, B.D.O. Anderson, Asymptotically fast solution of Toeplitz and related systems of linear equations, *Linear Algebra Appl.* 34: 103-116 (1980).

[3] R. Brent, F. Gustavson, D. Yun, Fast solution of Toeplitz systems of equations and and computation of Padé approximants, *J.Algorithms* 1: 259-295 (1980).

[4] M. Van Barel, G. Codevico, and G. Heinig, A superfast solver for real symmetric Toeplitz systems using real trigonometric transformations, *Proceedings of the Sixteenth International Symposium on Mathematical Theory of Networks and Systems*, pp. 1-5, 2004

[5] F. de Hoog, A new algorithm for solving Toeplitz systems of equations, *Linear Algebra Appl.* 88/89: 123-138 (1987).

[6] G. Heinig Transformation Approaches for Fast and Stable Solution of Toeplitz Systems and Polynomial Equations, *Proceedings of RAAM Workshop*, Kuwait University, May 1996, 223–238.

[7] G. Heinig, Chebyshev-Hankel matrices and the splitting approach for centrosymmetric Toeplitz-plus-Hankel matrices, *Linear Algebra Appl.* **327**, 1-3 (2001), 181–196.

[8] G. Heinig, P. Jankowski, Fast algorithms for the solution of general Toeplitz systems, *Wiss. Zeitschr. TU Karl-Marx-Stadt*, **32**, 1 (1990), 12–16.

[9] J.C. Mason, Chebyshev polynomials of the second, third and fourth kind in approximation, indefinite integration, and integral transforms, *J.of Comp.and Appl.Math.* 49 (1993), 169-178.

[10] M. Morf, Doubling algorithms for Toeplitz and related equations, *IEEE Internat.Conf. on Acoustics, Speech and Signal Processing, Denver 1980*, pp.954-959.

[11] V. Y. Pan, *Structured Matrices and Polynomials*, Birkhäuser Verlag, Boston and Springer–Verlag, New York, 2001.

[12] M. Stewart, A superfast Toeplitz solver with improved numerical stability, *SIAM J. Matrix Anal. Appl.*, 25 (3) (2003), 669-693.

[13] M. Tasche, Fast algorithms for discrete Chebyshev-Vandermonde transforms and applications, *Numerical Algorithms* 5: 453-464 (1993).

[14] M. Van Barel, G. Heinig, P. Kravanja, A stabilized superfast solver for nonsymmetric Toeplitz systems, *SIAM J. Matrix Anal. Appl.*, 23 (2) (2001), 449–510.