Twenty-five researchers (which appeared a suitable number) participated in our group, and 12 papers were discussed. In order to make the discussion easier and more efficient, we first decided to organize 4 sessions of 3 papers, each session being devoted to a particular theme, namely:

- embodied cognition (Ferrara, Edwards, Leron)\(^1\)
- role of metaphors (Attorps, Orfanos & Kalavassis, Matheron)
- geometry (Parzysz, Potari et al., Robotti)
- young children (Bills, Priolet & Régnier, Tortora & Iannece)

For people not familiar with the field, it may perhaps be of some use to begin with some words about embodied cognition and metaphors. The notion of ‘embodied cognition’ has been developed by Lakoff & Nuñez from two main ideas:

- concepts are structured by the brain and by the nature of the body
- abstract notions are constructed from basic conceptual mechanisms including:
  - conceptual metaphors
  - image schemas

Concerning ‘conceptual metaphors’, Lakoff & Nuñez distinguish three main types (Ferrara):

\(^1\) Each paper will be identified by the name of its author(s) written in italics.
– grounding metaphors
– linking metaphors
– redefining metaphors

But within our group the word ‘metaphor’ was used in a broad sense, which embraced notions such as:
– the usual linguistic meaning
– ‘ tool ’
– ‘ representation ’
– ‘ image ’
– ‘ analogy ’
– ‘ model ’
– ...

We can take ‘metaphor’ to mean the transfer of a ‘sign’ or a set of ‘signs’ which are proper to one domain (source) to another domain (target) (Potari et al.). In fact, metaphorical language is a ‘natural’ way of speaking about things. Examples:
– ‘A is B’
– ‘A looks like B’, ‘A is like B’ : similes
– metonymy...

BUT...

• ‘ Metaphor does not reside in words; it is a matter of thought. Metaphorical linguistic expressions are surface manifestations of metaphorical thought. ’ [Lakoff & Núñez 1997] (Bills)

• Metaphoric thought implies a mapping between cognitive structures. The use of metaphoric language can give us an indication of the metaphoric thought; in particular, metaphoric language associated with pedagogic representations can indicate the influences on the thinking of learners (Bills).

• In the education of young children, metaphoric thought can provide a link between ‘natural’ and ‘scientific’ concepts.

• Learners’ concept images need to be enriched through activities which can help them to develop awareness of the differences and commonalities in a variety of representations in order to develop and gain control of their mathematical thinking.
‘The general law of development states that awareness and control are peculiar properties of the higher stage of development of any function. They come late into being and are necessarily preceded by a stage of unconscious and unintentional functioning of any form of mental activity. In order to be aware of something, we must already possess that thing. In order to control something, we must have at our disposal the thing on which to exert our will.’ [Vygotsky 1934] (Tortora & Iannece)

Development of the learner’s conceptions may be enhanced through conversion between different types of representations in different contexts [Duval 1995] (Priolet & Régnier, Robotti)

The teacher has a role in engineering the social context for developing the awareness of different representations (Priolet & Régnier, Parzysz)

Representations may be imposed by the teacher or can result from negotiation between teacher and learners (Priolet & Régnier, Tortora & Iannece).

During the sessions, several questions posed by current research were discussed:

- What is the nature of the link between ‘informal mathematics’ and ‘formal mathematics’? (Edwards, Leron)
- How can metaphorical thinking foster the transition from perception to theory? (Ferrara)
- What is the role of memory and previous experiences in the use of metaphors? (Matheron, Attorps)
- How can we decide which metaphors are ‘good’?
- How can students’ and teachers’ conceptions be combined in a meaningful way?

And, since metaphors cannot match all the properties of the source with those of the target:

- How can teachers become aware of the possibilities and limits of metaphors? (Parzysz, Orfanos & Kalavassis)
- How can teachers manage students’ grasping of pertinent similarities and be aware of the limits? (Orfanos)
- How to make metaphorical discourse effective? (Robotti)

Finally, we left CERME 3 with some ideas for future research, and more especially the need for comparative studies which might identify different practices in various areas (problem solving, calculation, graphical representations, geometry…) through:

- informal exchange of ideas
– funded projects.

N.B.: An e-mail list (or web-site) for « Metaphors » (opened to all) will be created\(^2\) for:

– data bank of metaphors
– bibliography
– summaries (in English) of papers published by the members of the list.

List of contributions

List of Thematic Groups

\(^2\) Laurie Edwards volunteered to manage it.