METACOGNITIVE DISCOURSE IN MATHEMATICS CLASSROOMS

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Abstract: The purpose of the present study was to compare students' mathematical discourse under two conditions: cooperative learning embedded within metacognitive instruction (MT) and cooperative learning with no metacognitive instruction (CL).

Participants were 112 eighth graders who studied in four heterogeneus classrooms. Data were video-taped and analyzed by using quantitative and qualitative methods. Discourse analaysis indicated different characteristics under these two conditions. Students who were exposed to the metacognitive instruction within cooperative settings were better able than their counterparts in the CL condition to express their mathematical idea. Their mathematical discourse was more fluent and involved richer mathematical concepts. In addition, their discourse involved more frequently self-regulating behaviors (e.g., prove, check) than students who studied in cooperative settings with no metacognitive instruction. The practical implications of the study will be discussed on the conference.

Research in the area of mathematics emphasizes the importance of discourse as an integral part for the success of doing mathematics (The National Council of Teachers of Mathematics, 2000). The discourse in mathematics classrooms includes at least two factors: *mathematical discourse* and *metacognitive discourse.* Mathematical discourse includes the abilities to construct mathematical conjectures, develop and evaluate mathematical arguments, and select and use various types of representations. Metacognitive discourse refers to using self-regulating behaviors that are vital prerequisite for the successful acquisition of knowledge in school and beyond. In particular they are important with respect to lifelong learning (PISA.OECD, 2000).

Researchers (e.g., Schoenfeld, 1985; Mevarech & Kramarski, 1997; Kramarski, Mevarech & Arami, 2002) note that features of self-regulated behaviors can be learned through practice and reinforcement. Participation in reflective discourse in which mathematical activity is objectified can be an explicit topic of conversation.

While there is a growing consensus among researchers (e.g., Cobb, 1995; NCTM, 2000) about the positive effects of mathematical discourse on students' achievement, the question about under what conditions mathematical discourse has these effects is still open**.** This paper aims to contribute to efforts to understand how constructing mathematical meanings may emerge out of discourse substance.

Much current research in mathematical learning has been focusing on the facilitating role played by *cooperative learning* in mathematical discourse. For positive outcomes to occur research (e.g., Schoenfeld, 1992; King, 1994) has suggested that, small-group activities must be structured to maximize the chances that students will engage in mathematical discourse such as: questioning, elaboration, explanation, and other verbal communication in which they can express their ideas, monitor their actions and through which the group members can give and receive feedback.

The method of Mevarech & Kramarski, (1997) , called ***IMPROVE***, emphasizes the importance of providing each student with the opportunity to construct mathematical meaning by involving themselves in *metacognitive discourse*. The ***IMPROVE*** method is based on self-questioning via the use of metacognitive questions that focus on: (a) *comprehending* the problem (e.g.,"What is the problem all about"?); (b) constructing *connections* between previous and new knowledge (e.g., "What are the *similarities/differences* between the problem at hand and the problems you have solved in the past? and why?"; (c) using appropriate problem solving *strategies* (e.g., "What are the strategies/tactics/principles appropriate for solving the problem and why?"; and in some studies, (d) *reflecting* on the processes and the solution (e.g., "What did I do wrong here?"; "Does the solution make sense?").

Generally speaking, researchers (e.g., Schoenfeld, 1992; Mevarech & Kramarski, 1997; Kramarski, Mevarech, & Arami, 2002) reported positive effects of cooperative-metacognitive instruction on students' mathematical achievement. In particular, in the ability to explain mathematical reasoning. There is also evidence showing that the effects of metacongitive instruction employed in mathematics and foreign language classrooms on mathematics achievement were more positive than the effects of metacognitive instruction employed only in mathematics classrooms, which in turn were more positive than no metacognitive instruction (Kramarski, Mevarech, & Liberman, 2001). Yet, effects of IMPROVE method have been documented by data analyses based on paper-and-pencil measurements with no discourse analysis. There is reason to suppose that students who are exposed to metacognitive discourse are expected to be better able to reflect on solution processes than students who are not exposed to such discourse. Providing metacognitive instruction within cooperative settings would enrich the mathematical discourse and by that would profit students' constructing meanings of mathematical ideas.

The purpose of the present study was to investigate the differential effects of metacognitive instruction on mathematical discourse. In particular, the study compares two instructional methods: cooperative learning embedded within metacognitive instruction (MT), and cooperative learning without metacognitive instruction (CL).

To address that issue, the present study was conducted in two stages: in the first stage students were asked to solve cooperatively a task and their written discourse was assessed. In the second stage we analyzed videotaped oral discourse transcript from one group of each learning condition (MT vs. CL). Both groups were randomly selected from the teams that solved correctly the task.

Method

Participants

The study included 28 groups of eight grade students (112 boys and girls) who studied in mathematics lessons the graph unit in 4 heterogeneous classrooms selected from two junior high schools. Fourteen groups (N=56) participated in each learning condition. Each group consisted of four students: one low, two middle and one high achiever.

The students participated two month before in a large scale study which investigated effects of different learning conditions on graphing learning (Kramarski and Mevarech, in press). One-way analysis of variance (ANOVA) indicated no significant differences between conditions prior to the beginning of the study ($\underline{F}(1,54)=2.36$; $\underline{p}>.05$; $\underline{F}(1,54)=.21$; $\underline{p}>.05$, respectively for graph interpretation and graph construction).

Treatment

Students studied in mathematics lessons under one of two conditions: Cooperative learning embedded within metacognitive training (MT) and cooperative learning with no metacognitive training (CL).

The MT condition: The metacognitive instruction was based on the **IMPROVE** method using metacognitive questions. Regarding graph interpretation, the *comprehension questions* guided students to interpret the graph on both the local-to-global dimension and the quantitative-to-qualitative dimension (Leinhardt et al., 1990). The comprehension questions included the following: *What does the x-axis represent? What does the y-axis represent? What the graph shows? and What are the specific points on the graph?* Students had to explain their mathematical reasoning for each response. To assist students in using the comprehension questions, students used the acronym ***DATA***: ***D***escribe the x-axis and the y-axis; ***A***ddress the units and the ranges of each axis; ***T***ell the ***T***rend(s) of the graph or parts of the graph; and ***A***nalyze specific points on the graph.

The s*trategic questions* referred to strategies that students could use in interpreting the graphs. Strategies could refer to adding steps to the graph in order to calculate the slope, to using data-tables, or to referring to the algebraic representation of the graph.

The *connection questions* referred to questions that guided students to find similarities and differences between the graph at hand and graphs they had already interpreted or to compare different intervals on the same graph. For example: "How is this problem/task different from/ similar to what you have already solved?. Please explain your reasoning.

The *reflection questions* were designed to prompt students to *reflect* on their understanding and feelings during the solution process (e.g., ""Can I use another approach for solving the task?").

The metacognitive questions were printed in the Students' Booklets and Teacher Guide. The metacognitive questions were used by each individual student when his/her turn arrived to solve a problem/task aloud, by the group as a whole in the mathematical discourse, and in writing. In addition to the students, also the teachers modeled the use of metacognitive questioning when she introduced the new concepts to the whole class, reviewed the lesson at the end of the class, and provided help in the small groups.

The studying in groups was implemented as follows: each student, in his/her turn, read the task aloud and tried to solve/analyze it. Whenever there was no consensus, the group discussed the issue until a disagreement was resolved. Students were encouraged to talk about the task, explain it to each other, and approach it from different perspectives.

The CL condition: Under this condition, students studied in small groups without using the metacognitive questions. Each student in his or her turn read the task aloud and tried to solve it. When he or she failed to solve the task or when students did not agree upon the solution, the team discussed the task until consensus was achieved. When all team members agreed on a solution, they wrote it down in their notebooks. When none of the team members knew how to solve the task, they asked for teacher help. As indicated, the CL students were exposed to the same tasks as the ML students.

Measuresments and procedures

Two group problem solving tasks were implemented: an interpretation task and a construction task. In order to reduce background noise during solving the tasks, each team was videotaped separately in a quiet room in the school. The problem solving session lasted for approximately 30 minutes. The written and oral discourse of the groups were analyzed.

Written Group Problem Solving Discourse

<u>Graph Interpretation Task</u>: The graph below (Figure 1) represents the income of two companies between five years.

PART 1: Was the change-rate in the income of Company B *greater than/ smaller than/ equal* to the change-rate of Company A? Please explain your reasoning.

PART 2: After three years, did the change-rate in the companies' incomes become different? Please explain your reasoning.

Insert figure 1

Scoring: Each item, received a score of either 1 (correct answer/explanation) or 0 (incorrect answer). A total score ranging from 0 to 4. In addition the mathematical argumentations were categorized into three categories: Logic-Formal; Numerical-Computational; and Drawing explanations.

- Logic-Formal: based on logic-mathematical arguments (e.g., "The change-rate of line A is greater because its slope is steeper than that of line B").
- Numerical-Computational: explanations are based on numerical computations or the algebraic formula (e.g., "The change-rate of line A is _ and the change-rate of line B is 1/3");
- Drawing:: refer to lines or other kinds of visual symbols added to the graph (e.g., "Adding one-unit steps to the graph and calculating the change rate by using the steps").

Two judges who are experts in mathematics education analyzed students' explanations. Inter-judge reliability coefficient was .88.

Graph Construction Task: The following three bottles are filled with water during 5 minutes. Construct a graph that presents the rate of the filling of water in each bottle.

Scoring: A team received a score of 0 (incorrect) to 2 (full answer) for each graph that presents the rate of the entrance of water in each bottle. A total score ranging from 0 (incorrect) to 6 (full answer).

Oral Group Problem Solving Discourse

Qualitative Analysis was conducted on students' oral group problem solving discourse. Their mathematical discourse was classified into four criteria: Vocabulary; Fluency; Strategies Explanations; and Metacognitive expressions.

Results

Analyzing Written Group Problem Solving Discourse

Table 1 presents the frequencies (percent in parentheses) of groups providing correct answers on graph interpretation, verbal explanations and graph construction by treatment and tasks.

Insert Table 1& 2 about here

The findings indicate that the MT groups outperformed the CL groups on graph interpretation task and on mathematical explanations as well as on graph construction. Table 2 indicates that on the second part of the task students under both conditions were very often involved in Logic – Formal explanations. Furthermore, the MT students were more flexible in their explanations, they often used more then one argument in their explanations. In particular they based their arguments on drawing steps.

Analyzing Oral Group Problem Solving Discourse

Comparing the two discourses shows significant differences in the quality of mathematical *discourse* of each group, as follows:

(a) *Vocabulary*: The mathematical vocabulary of the MT group was richer than that of the CL group. The former based their arguments on concepts such as: rate, change-rate, line, unit, and the ”height of the step”. They looked for a formal definition, elaborated it, and applied it correctly as the solution evolved. The CL group based their arguments mainly on computations and mathematical terms were interpreted on the basis of everyday knowledge.

(b) *Fluency*: The discourse of the MT group was longer than that of the CL group . The CL group, although exposed to the same curriculum, did not use in their written response explain their line of reasoning accurately.

(c) *Explaining strategies* : Generally speaking, the MT group did the task by interpreting the graph qualitatively, whereas the CL group interpreted the graph quantitatively. The MT students used a richer repertoire of strategies than CL group. The former developed the need to support their answers by formal mathematical arguments. They, for example, looked for a formal definition, used the textbook to reflect on their approach, and checked their mathematical reasoning by adding drawings to the graph. Conversely, the CL group, used a (wrong) visual strategy , and then gradually moved into computations.

(d) Metacognitive expressions: The mathematical discourse in the MT condition was led by encouraging peers to “prove” and “check”. This is in contrast to the CL group who from the very beginning and through out the entire discourse referred to specific context points on the graph.

Discussion

The study investigates the differential effects of cooperative-learning with or without metacognitive instruction (MT vs. CL) on mathematical discourse. Although the both methods focus to promoting mathematical discourse it was found that students that were exposed to cooperative-learning with metacognitive discourse (MT) facilitated positive effects on graph interpretation and graph construction more than students that were exposed to cooperative learning without metacognitive discourse (CL). Analysis of the discourse provided insights into the nature of mathematical discourse and into the ways in which the characteristics of that talk can have differential effects for each interlocutor with respect to the mathematics that emerges within the interactions. It seems that using metacognitive questions enriched the "*taken-as-shared"* mathematical meaning in an interactive way (Cobb, 1995). An support for this hypothesis comes from the fact relates to the quality of the discourse within the two cooperative conditions. The findings indicate that whereas, students under the MT condition gave more often elaborated explanations the CL students are more often involved in technical communication. It might be that differences in the types of communication in which the children engaged influenced the learning opportunities that arose for them. These findings support other studies (e.g., Webb, 1991) who showed that asking students to answer *why* questions during the solution processes helped them to elaborate and retain information. As students explain and justify their thinking, and as they challenge the explanations of their peers and teachers, they are also engaging in clarifying of their own thinking and becoming owners of "knowing" (Lampert, 1990).

The practical implications of the study and suggestions for future research will be discussed on the conference.

References

Mevarech, Z.R. & Kramarski, B. (1997). IMPROVE: A multidimensional method for teaching mathematics in heterogeneous classrooms. *American Educational Research Journal, 34* (2), 365-395.

National Council of Teacher of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

Table 1: Frequencies (percent in parentheses) of groups providing correct answer on graph interpretation, verbal explanations and graph construction by treatment.

	MT n=14 groups	CL n=14 groups	$\_^2$

PART 1			
Graph Interpretation	14 (100)	11 (78.6)	3.36; df=1; p<.06
Verbal Explanations	14 (100)	7 (50.0)	6.08; df=1; p<.01
PART 2			
Graph Interpretation	14 (100)	8 (57.1)	7.64; df=1; p<.02
Verbal Explanations	13 (92.9)	8 (57.1)	6.37; df=1; p<.04
Graph Construction			
Full answer	10 (71.4)	3 (21.4)	8.59; df=1; p< .01
Partial answer (at least one correct graph)	4 (28.6)	7(50.0)	

[1] Note: Percents in each category were calculated by dividing the number of groups using that category with the total number of groups in that treatment.

Table 2: Frequencies[1] (percent in parentheses) of groups providing different kinds of correct arguments on graph interpretation by treatment

	MT n=14 groups	CL n=14 groups	$\_^{2}$
PART 1			
Kind of Arguments			6.67; df=2; p<0.03
Logic-Formal	12 (85.7)	3 (21.4)	
Numerical-Computational	3 (21.4)	5 (35.7)	
Drawing	7 (49.7)	----	
More than one argument	5 (35.7)	2 (15.4)	
PART 2			
Kind of Argument			7.17; df=2; p<0.02
Logic-Formal	10 (71.4)	7(49.7)	
Numerical-Computational	1 (7.1)	1 (7.1)	
Drawing	5 (35.7)	----	
More than one argument	5 (35.7)	1 (7.1)	

[1] Note: Percents in each category were calculated by dividing the number of groups using that category with the total number of groups in that treatment.

Figure 1: