Research report

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Oberwolfach, 2010

In the following, all structures expands an integral domain.

Definition 1. A structure M is geometric if the algebraic closure is a pregeometry in all M' elementarily equivalent to M ([DMS08] observe that in this case M eliminates the quantifier $\exists^\infty$).

We define a generalisation: structures with an "existential" matroid. The main examples are superstable structures of U-rank a power of ω and d-minimal structures.

Definition 2. A d-minimal structure is a structure M with a definable Hausdorff topology, such that every definable subset X of M is the union of an open set and finitely many discrete sets (where the number of discrete sets does not depend on the parameters of definition of X), plus some additional conditions.

O-minimal structures, p-adic fields, and algebraically closed valued fields are d-minimal (and also geometric).

On a structure there can be at most one existential matroid. Ultraproducts of geometric structures, while not geometric in general, do have an unique existential matroid. An existential matroid on M can be extended in a canonical way to a closure operator on M^{eq} (see [Gagelman05] for the case when M is geometric).

A dimension function on M is a function from M-definable sets to the natural numbers, satisfying some natural conditions [Dries89, MS07]. There is a canonical correspondence between dimension functions and existential matroids.

Generalising previous results in [Dries98], we study dense closed pairs of structures with an existential matroid. More precisely, $X \subseteq M$ is dense if X intersects every definable subset of M of dimension 1 (when M is geometric,

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the above notion was already considered in [Macintyre75]). Given T the theory of a structure with an existential matroid, let T^d be the theory of pairs (B, A) such that $B \models T$ and A is a dense and closed subset of B. Then, T^d is consistent and complete. Moreover, the models of T^d also have an existential matroid, the "small closure": $b \in scl(X)$ if b is in the closure of $A \cup X$. We extend the above result to dense tuples of structures:

Theorem 3. *Let T be the theory of a structure with an existential matroid, Define T^{nd} be the theory of tuples $A_0 \prec A_1 \prec \ldots \prec A_n \models T$, such that each A_i is closed in A_n, and A_0 is dense in A_n. Then, T^{nd} is consistent and complete.*

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