

# BRAIDS AND THEIR RAMIFICATIONS

Configuration Spaces, Arrangements, Mapping-Class Groups, 3-Manifolds

Cortona, Italy. May 20-26, 2007

## Abstracts

**Anders Björner** (KTH Stockholm)

**Title:** On the shape of Bruhat intervals.

Abstract: Let  $(W, S)$  be a crystallographic Coxeter group. In particular, it could be a finite or affine Weyl group. For  $w \in W$ , let  $f_i$  denote the number of elements of length  $i$  below  $w$  in Bruhat order. We are interested in properties of the number sequence  $f_0, f_1, \dots, f_{l(w)}$  and relations to other structural features of Bruhat intervals  $\{x \in W \mid x \leq w\}$ .

It is shown that

- (1)  $f_i \leq f_{l(w)-i}$ , for all  $i \leq l(w)/2$ ,
- (2)  $f_0 \leq f_1 \leq \dots \leq f_{l(w)/2}$ ,
- (3) if  $W$  is finite and  $w$  is of sufficient length (with respect to  $k$ ):  $f_{l(w)-k} \leq f_{l(w)-k+1} \leq \dots \leq f_{l(w)}$ .

Parts (1) and (2) are true also for  $W$  modulo a parabolic subgroup.

(This is joint work with T. Ekedahl.)

**Christian Blanchet** (Université de Bretagne-Sud)

**Title:**  $sl(N)$  link homology from trivalent TQFT.

Abstract: For each integer  $N$ , we construct a TQFT for planar trivalent graphs and trivalent surfaces. Following Khovanov construction for  $sl(2)$  and  $sl(3)$ , we use this TQFT to define an homology of links. This homology is defined over integer and should be rationally equivalent to Khovanov-Rozansky  $sl(N)$  link homology. We will consider functoriality properties and Lee-Rasmussen type degeneracy.

**Michel Boileau** (Université de Toulouse III)

**Title:** Tits alternative and  $PD(3)$  groups.

Abstract: We will discuss the existence of non-abelian free groups in  $PD(3)$  groups. We will show in particular that a coherent  $PD(3)$  group of rank at least 4 always contains a non-abelian free group. This is a joint work with Steve Boyer.

**Kenneth Bromberg** (University of Utah)

**Title:** Combinatorial distance and hyperbolic distance between simple closed geodesics.

Abstract: We show that the distance between closed geodesics in the hyperbolic 3-manifolds is bounded below by a linear function of the distance between the corresponding vertices in the complex of curves. We will also describe some applications of this result.

**Ryan Budney** (University of Victoria)

**Title:** De-looping embedding spaces.

Abstract: I will survey what is known about the homotopy-type of the space of smooth embeddings of one sphere in another,  $Emb(S^j, S^n)$ . Denote the space of embeddings of  $R^j$  in  $R^n$  which agree with a fixed linear embedding outside of a fixed ball by  $K_{n,j}$ , the ‘long’ embedding space. One basic result is that  $Emb(S^j, S^n)$  fibres over a Stiefel manifold with fibre  $K_{n,j}$ . Let  $K_{n,j}^+$  denote the space of long embeddings of  $R^j$  in  $R^n$  where each embedding comes with a trivialization of its normal bundle. An observation made four years ago is that  $K_{n,j}^+$  admits the action of the operad of little  $(j+1)$ -cubes. This makes  $K_{n,j}^+$  into a  $(j+1)$ -fold loop-space provided  $n - j > 2$ . Questions related to the structure of  $K_{n,j}^+$  as an object over the operad of  $(j+1)$ -cubes have motivated much of my recent research.  $K_{3,1}^+$  turns out to have the homotopy-type of  $K_{3,1}xZ$ , and  $K_{3,1}$  is a free 2-cubes object over the space of prime long knots. The proof of this is a detailed relationship between the JSJ-decomposition of knot complements and the action of the operad of 2-cubes on  $K_{3,1}$ . This led to an essentially complete ‘computation’ of the homotopy-type of  $Emb(S^1, S^3)$ , which I will outline. Some other developments will be mentioned, such as a generalized Litherland-spinning construction to generate all the Haefliger spheres:  $\pi_0 Emb(S^j, S^n)$  for  $n - j > 2$ , and the computation of the first non-trivial homotopy-groups of  $K_{n,j}$  for  $2n - 3j - 2 > 0$ .

**Filippo Callegaro** (Università di Pisa)

**Title:** The  $K(\pi, 1)$  problem for the affine Artin group of type  $\tilde{B}_n$  and its cohomology.

**Abstract:** We prove that the complement of the affine complex arrangement of type  $\tilde{B}_n$  is a  $K(\pi, 1)$  space. Hence we compute the cohomology of the affine artin group of type  $\tilde{B}_n$  with coefficients over the module  $\mathbb{Q}[q^{\pm 1}, t^{\pm 1}]$  where the first  $n$ -standard generators act by  $(-q)$  multiplication and the last generator acts by  $(-t)$ -multiplication. Such representation generalizes the analog 1-parameter representation that corresponds to considering the structure of bundle over the complement of the discriminant hypersurface and the monodromy action of the associated Milnor fibre. As a corollary we derive the cohomology of the affine artin group of type  $\tilde{B}_n$  with trivial coefficients.

This is a joint work with Davide Moroni and Mario Salvetti

**Dick Canary** (University of Michigan)

**Title:** Introductory Bumponomics: untouchable points in boundaries of deformation spaces of hyperbolic 3-manifolds.

**Abstract:** In this talk, we study the space  $AH(M)$  of all (marked) hyperbolic 3-manifolds homotopy equivalent to a fixed compact 3-manifold  $M$  (with boundary.) The topology of the interior of  $AH(M)$  is quite simple and has been well-understood since the 1970's. However, in the last decade it has become clear that the global topology of  $AH(M)$  is quite complicated.

In this talk, we will survey the history and discuss recent joint work with Brock, Bromberg and Minsky, which shows, in many cases, that the topology is well-behaved at "most" points in the boundary of  $AH(M)$ .

**Alessia Cattabriga** (Università di Bologna)

**Title:** Extending homeomorphisms from punctured surfaces to handlebodies.

**Abstract:** Let  $H_g$  be a genus  $g$  handlebody and set  $T_g = \partial H_g$ . In this talk we deal with  $\mathcal{E}_{2n}^g$ , the subgroup of  $\text{MCG}_{2n}(T_g)$  consisting of the isotopy classes of homeomorphisms of  $T_g$  which admit an extension to the handlebody keeping fixed the union of  $n$  disjoint properly embedded trivial arcs. The main motivation for studying this group is given by its connection with  $(g, n)$ -decompositions of knots and links in 3-manifolds.

In the first part of the talk we introduce the notion of  $(g, n)$ -decomposition and explain the role of  $\mathcal{E}_{2n}^g$  in this contest. Then we describe a finite set of generators for this group and, finally, we give a sketch of the proof.

**Young-Eun Choi** (Penn State Altoona)

**Title:** Comparison of Teichmueller geodesics and lines of minima.

Abstract: Teichmueller geodesics and lines of minima are infinite paths in Teichmueller space that share common properties. Given two measured foliations on a surface, we compare the behavior of the associated Teichmueller geodesic and line of minima. In particular, we show that the curves which become short along the geodesic coincide with those that become short along the line of minima. We find that the line of minima can deviate arbitrarily far from the geodesic, but by analyzing the change in length of short curves and the structure of their complements, we show that lines of minima are quasi-geodesics. (Joint work with Kasra Rafi and Caroline Series)

**Dan Cohen** (Louisiana State University)

**Title:** Motion planning in tori.

Abstract: Let  $X$  be a path-connected topological space. Viewing  $X$  as the space of configurations of a mechanical system, the motion planning problem from robotics consists of constructing an algorithm which takes as input pairs of configurations  $(A, B)$  in  $X \times X$ , and produces a continuous path in  $X$  from the initial configuration  $A$  to the terminal configuration  $B$ . For most spaces  $X$ , it is not possible to construct a globally continuous motion planning algorithm. So one divides the product space  $X \times X$  into “local domains” over which the motion planning problem can be solved continuously. The minimal number of local domains required is called the topological complexity of  $X$ . We compute this number for several natural subcomplexes of the  $n$ -dimensional torus.

This is joint work with G. Pruidze.

**Fred Cohen** (University of Rochester)

**Title:** On spaces of representations.

Abstract: This lecture gives topological properties of spaces of homomorphisms  $Hom(\pi, G)$  together with certain elements in the space of representations. The group  $G$  is assumed to be a Lie group, not necessarily compact, and  $\pi$  is a discrete group. Special emphasis on groups “similar” to braid groups will be given. This lecture is based on two joint projects one with Alex Adem and one with Stratos Prassidis, Jon Lopez and Marston Conder.

**Patrick Dehornoy** (Université de Caen)

**Title:** The alternating normal form for positive braids.

Abstract: We describe a new normal form for elements of the braid monoid  $B_n^+$ . It is reminiscent of the greedy normal form but, instead of considering the maximal divisor of the considered braid that is a divisor of Garside's fundamental braid  $\Delta_n$ , one takes into account the maximal divisor that lies in  $B_{n-1}^+$ . The resulting normal form turns out to be connected with the braid order in a deep way, improving our understanding of the latter significantly.

**Graham Denham** (University of Western Ontario)

**Title:** Right-angled Artin groups and monomial ideals.

Abstract: A combinatorial construction originating in work of Davis and Januszkiewicz gives a variety of interesting topological spaces: these include classifying spaces for certain Coxeter and Artin groups, complements of coordinate subspace arrangements, and also the moment-angle complexes of Buchstaber and Panov.

The cohomology of such spaces can often be seen in terms of combinatorial commutative algebra. I will give some examples. In particular, resolutions of monomial ideals over exterior algebras describe cohomology of certain subgroups of right-angled Artin groups. In particular, this gives additional information about the homological finiteness properties of such subgroups.

**Alexandru Dimca** (Université de Nice - Sophia Antipolis)

**Title:** Characteristic varieties and constructible sheaves.

Abstract: In this talk we explain the relation between the irreducible components of the first characteristic variety of a smooth quasi-projective complex variety and the singular sets of some naturally associated constructible sheaves.

**Eva Feichtner** (Stuttgart University)

**Title:** Nested set complexes in geometric combinatorics and beyond.

Abstract: Nested set complexes - the combinatorial core structure of De Concini-Procesi wonderful arrangement models - have figured prominently at various places throughout geometric combinatorics and most recently in tropical geometry. In this talk we will survey the history and the manifold appearances of these truly wonderful gadgets.

**Cameron Gordon** (University of Texas)

**Title:** Heegaard genus and Dehn filling.

Abstract: The context of the talk is the general question of the behavior of Heegaard genus under two distinct Dehn fillings on a 3-manifold with torus boundary; more specifically we consider the case where one of the fillings is  $S^3$ . The result is that there is a function  $w(g)$  such that if  $K$  is a hyperbolic knot in  $S^3$ , and some non-integral Dehn surgery on  $K$  gives a non-Haken manifold of Heegaard genus  $g$ , then the tunnel number of  $K$  is at most  $w(g)$ . We will also discuss the special case  $g = 2$  and its relevance to the conjecture that any Seifert fibered Dehn surgery on a hyperbolic knot in  $S^3$  must be integral. (Joint work with Ken Baker and John Luecke.)

**Anthony Henderson** (University of Sidney)

**Title:** The cohomology of real De Concini-Procesi models of Coxeter type.

Abstract: To any hyperplane arrangement one can associate a De Concini-Procesi model by blowing up the intersections in a prescribed way. In the case of the braid arrangement, this leads to the moduli space of stable genus 0 curves with marked points. Some remarkable results about the cohomology of the real points of the latter variety, including a formula for the character of the representation of the symmetric group on each rational cohomology group, have been discovered recently by Etingof, Henriques, Kamnitzer, and Rains. I will discuss how these results can be generalized to other finite Coxeter types. In types B and D this involves the homology of sub-posets of the signed partition lattice. This is joint work with Eric Rains (UC Davis).

**Craig Hodgson** (University of Melbourne)

**Title:** Enumeration and classification of knotted graphs using hyperbolic invariants.

Abstract: I will describe work with Chiodo, Heard, Saunderson and Sheridan on knotted trivalent graphs in the 3-sphere; and work with Heard, Martelli and Petronio on knotted trivalent graphs in general closed 3-manifolds.

Graphs are enumerated using two different methods: the first based on Conway's approach to enumerating knot projections, the second using triangulations. The graphs are then classified using hyperbolic invariants computed using D. Heard's computer program Orb. Applications to the enumeration of low volume hyperbolic 3-orbifolds will also be mentioned. If time permits, I will conclude with a brief demonstration of Orb.

**Toshitake Kohno** (University of Tokyo)

**Title:** Hyperbolic volumes, configuration spaces and iterated integrals.

Abstract: Using ideas due to Aomoto, we express the volume of a simplex in spherical or hyperbolic space by means of iterated integrals of logarithmic forms on the configuration spaces. We show that the volume functions satisfy the differential equations arising from nilpotent connections and describe asymptotic behavior of such functions.

**Feng Luo** (Rutgers University)

**Title:** Rigidity of Polyhedral surfaces.

Abstract: Classical differential geometry deals with smooth surfaces and Riemannian metrics. In contrast, a polyhedral surface, such as a tetrahedron, is a surface composed of Euclidean (or spherical, hyperbolic) triangles. This talk discusses the geometry of polyhedral surfaces. One of the main problems on surface geometry is to understand the relationship between curvature and metric. The metric-curvature relation for polyhedral surfaces is governed by the cosine law. We will show you how derivative of the cosine law implies many rigidity phenomena about the polyhedral surfaces. Applications to the Teichmüller space of surfaces with boundary will also be discussed.

**Bruno Martelli** (Università di Pisa)

**Title:** Complexity and decompositions of PL-manifolds.

Abstract: We define for every PL compact manifold  $M$  of dimension  $n$  and any integer  $0 < k < n$  a non-negative integer  $c_k(M)$ , called the  $k$ -th complexity of  $M$ , which measures the “minimal complexity” of the  $k$ -th stratum of a cellularization of  $M$ .

For  $n = 2, 3$ ,  $c_k$  is equivalent to well-known invariants such as the Euler characteristic for surfaces, and the Heegaard genus and Matveev complexity for 3-manifolds. Concerning 4-manifolds,  $c_1$  measures the minimum number of 1-handles in a decomposition, and  $c_2$  the minimum number of vertices of a Turaev shadow.

We then focus here on the case  $k = 2$  and  $n = 4$ . The function  $c_2$  leads naturally to a notion of decomposition of any PL 4-manifolds into prime ones, and of prime 4-manifolds into blocks, very similar in nature to the prime and JSJ decomposition of 3-manifolds, with  $S^2 \times S^1$  playing the rôle of the torus  $S^1 \times S^1$ , and (a refined version of)  $c_2$  playing the rôle of Gromov norm. Both decompositions are defined by selecting (among all possible decompositions) only those who preserve the total complexity of the blocks.

Finally, we study the set of 4-manifolds having complexity zero. Pushing further the analogy with 3-manifolds, this set has many features in common with the set of graph manifolds.

**Alexander Mednykh** (University of Novosibirsk)

**Title:** Volumes of Polyhedra and their applications to Knot Theory.

Abstract: Volume calculations for polyhedra in the spaces of constant curvature is very old and difficult problem. The first result in this direction was obtained by Tartaglia (1494) who found a formula for volume of Euclidean tetrahedron. Now this formula is known as Cayley-Menger formula. The volumes for non-Euclidean biorthogonal tetrahedra were given by Lobachevsky (1836) and Schläfli (1858). Vinberg (1991) determined the volume for tetrahedron with at least one vertex on the infinity. The volume formula for the Lambert cube was obtained by Ruth Kellerhals (1989) and Mednykh and Derevnin (2001) in hyperbolic and spherical spaces, respectively.

Despite of this partial results, a formula for the volume of an arbitrary hyperbolic tetrahedron has been unknown until very recently. The general algorithm for obtaining such a formula was indicated by Hsiang (1988) and the complete solution of the problem was given by Yu. Cho and H. Kim (1999), J. Murakami and M. Yano (2001), A. Ushijima (2002).

In these papers the volume is expressed as an analytic formula involving 16 Dilogarithm or Lobachevsky functions whose arguments depend on the dihedral angles of the tetrahedron and on some additional parameter which is a root of some complicated quadratic equation with complex coefficients.

Just recently, it became known that very simple and beautiful formula for hyperbolic tetrahedron was discovered by Italian Duke Gaetano Sforza (1906). More precisely, the volume for hyperbolic tetrahedron is given by

$$\text{Vol}(T) = \frac{1}{4} \int_{A_0}^A \log \frac{c_A - \sqrt{-\det G} \sin A}{c_A + \sqrt{-\det G} \sin A} dA,$$

where  $G$  is Gram matrix of the tetrahedron,  $c_A$  is algebraic compliment of  $G$  with respect to dihedral angle  $A$ , and  $A_0$  is a root of the equation  $\det G = 0$ .

We discuss similar results for a wide class of polyhedra in the spaces of constant curvature. As application, we give explicit formulae for the Figure eight knot, Whitehead link, Borromean rings and others cone-manifolds.

**Stefan Papadima** (Institute of Mathematics, Bucharest)

**Title:** Cohomology jumping loci of algebraic varieties.

**Abstract:** I will describe several special features of characteristic and resonance varieties coming from smooth complex quasi-projective varieties. Related properties, together with applications, will also be discussed.

**Luis Paris** (Université de Bourgogne)

**Title:** Singular Hecke algebras, Markov traces, and link invariants.

**Abstract:** A *singular braid* is a braid which admits finitely many transversal double-points. The isotopy classes of singular braids on  $n$  strands form a monoid (and not a group) called the *singular braid monoid* and denoted by  $SB_n$ . Let  $\mathbb{K} = \mathbb{C}(q)$ . We define the *singular Hecke algebra*  $\mathcal{H}_q(SB_n)$  to be the quotient of the monoid algebra  $\mathbb{K}[SB_n]$  by the so-called Hecke relations:

$$\sigma_i^2 = (q - 1)\sigma_i + q, \quad 1 \leq i \leq n - 1,$$

where  $\sigma_1, \dots, \sigma_{n-1}$  are the standard generators of the braid group  $B_n \subset SB_n$ . Following the same approach as Jones for the non-singular Hecke algebras, we define the notion of a *Markov trace* on  $\{\mathcal{H}_q(SB_n)\}_{n=1}^{+\infty}$  and show that a Markov trace determines an invariant for singular links. Our main result is that the Markov traces form a graduated vector space  $TR = \bigoplus_{d=0}^{+\infty} TR_d$ , where  $TR_d$  is of dimension  $d + 1$ . The space  $TR_0$  is spanned by the Ocneanu trace, and, for  $d \geq 0$ ,  $TR_d$  is the space of traces defined on braids with  $d$  singular points. Thanks to this result, we can define a universal Markov trace which gives rise to a universal Jones-type invariant for singular links. This invariant turns to be a Laurent polynomial on 4 variables which can be computed by means of generalized skein relations. (Joint with Loïc Rabenda).

**Bernard Perron** (Université de Bourgogne)

**Title:** A new necessary and sufficient condition for the faithfulness of the Burau representation for  $B(4)$  (the braid group with 4 strings).

Abstract: It is known that the Burau representation for the braid group  $B(n)$  is unfaithful for  $n$  greater or equal to 5 (thank to Moody, Long-Patton, Bigelow) and faithful for  $n = 3$  (Magnus-Peluso). The remaining case  $n = 4$  is unsolved. Having in mind the "unitary" character of the Burau representation, I recently proved that the faithfulness for  $n = 4$  is equivalent to the fact that two explicit very "simple" matrices of  $SU(3)$  ("simple" meaning that they are stabilization in two different ways of matrices of  $SU(2)$ ) generate a free (non abelian) group. This pair of matrices is a priori much easier to handle than the one in Birman book (theorem 3.19). The question of freeness for the group generated by pairs of matrices has been intensively studied around 1960 for pairs of  $SO(3)$  which look very much like the pair I got (see DeGroot, Dekker, ..). Unfortunately their hypotheses are not fulfilled in our case. However I have strong hope to conclude in a near future.

**Katya Pervova** (Università di Pisa )

**Title:** Link invariants from Turaev-Viro state sums.

Abstract: We consider certain invariants of links in 3-manifolds obtained by a specialization of Turaev-Viro invariants of 3-manifolds. The specialization is based on presentations of pairs  $(M, L)$ , where  $M$  is a closed oriented 3-manifold and  $L \subset M$  is an oriented link, by triangulations of  $M$  such that  $L$  is a subset of the 1-skeleton. We consider some elementary properties of the invariants, including the behavior under the connected sums of pairs away and along links. These properties allow one to provide examples of links in the 3-sphere which have the same HOMFLY polynomial and the same Kauffman polynomial but distinct Turaev-Viro invariants (similar examples can be provided for the Alexander polynomial). We also provide examples of some specific calculations, in particular, we determine the values of Turaev-Viro invariants of order 5 for all torus knots. (Joint work with Carlo Petronio).

**Joan Porti** (Universitat Autònoma de Barcelona)

**Title:** Collapsing three manifolds.

Abstract: At the end of the proof of geometrization, there is a thin-thick decomposition for three manifolds, once the Ricci flow has run for sufficient long time. Perelman claims that the thin part is a graph manifold. There is a proof of that result by Shioya and Yamaguchi that uses heavy machinery on Alexandrov spaces. In this talk I want to give an alternative proof when the manifold is aspherical, which is sufficient for geometrization, based on simplicial volume and Thurston's geometrization of Haken manifolds. This is joint work with L. Bessières, G. Besson, M. Boileau and S. Maillot.

**Claudio Procesi** (Università di Roma I)

**Title:** From hyperplane arrangements to numerical analysis.

Abstract: To a list of vectors in real space are associated several analytic, topological and combinatorial objects. We shall discuss some relationship between associated splines and wonderful embeddings of the associated hyperplane arrangement.

**Richard Randell** (University of Iowa)

**Title:** Re-presenting and Representing the Pure Braid Groups.

Abstract: The pure braid groups have a classical presentation, going back to E. Artin, and classical representations, going back to Brauer and Gassner. These presentations are pleasant in that they arise from combing the pure braid, or from the iterated fibration of Fox-Fadell-Neuwirth, and they can thus be used to understand the groups as iterated semi-direct products of free groups. Here we look at other presentations, which arise from work of Margalit and McCammond, as well as from group presentations for real arrangements arising in (separate) work of Salvetti and the author. We consider these presentations in the light of minimal CW structures on the complement of the pure braid arrangement, including explicit such CW structures given by Yoshinaga. Finally, we use these presentations to give representations (described by H. Schroeder) of Lawrence-Krammer-Bigelow type for the pure braid groups.

**Dev Sinha** (University of Oregon)

**Title:** From the cohomology of the pure braid group to Hopf invariants.

Abstract: The cohomology of the pure braid group gives a canonical, combinatorially rich presentation for the Poisson and thus Lie co- operads. These models then may be used to further illuminate the interplay between commutative and Lie algebras, and in particular the interplay between the cohomology ring and homotopy Lie algebra of a space. This happens at both formal and geometric levels, leading to what is perhaps the definitive treatment of generalized Hopf invariants.

**Alex Suciu** (Northeastern University)

**Title:** Quasi-projective Bestvina-Brady groups.

Abstract: To each finite simple graph  $G$ , there is associated a right-angled Artin group  $A_G$ , and a Bestvina-Brady group  $N_G$ . An analysis of the cohomology jumping loci of these groups yields a complete characterization of those graphs  $G$  for which the groups  $A_G$  and  $N_G$  arise as fundamental groups of smooth, quasi-projective varieties. This leads to examples of quasi-projective groups which are not commensurable (even up to finite kernels) to the fundamental group of any aspherical, quasi-projective variety.

This is joint work with Alex Dimca and Stefan Papadima.

**Hiroaki Terao** (Hokkaido University)

**Title:** Chambers of Arrangements of Hyperplanes and Arrow's Impossibility Theorem.

Abstract: Let  $\mathcal{A}$  be a nonempty real central arrangement of hyperplanes and  $\mathbf{Ch}$  be the set of chambers of  $\mathcal{A}$ . Each hyperplane  $H$  makes a half-space  $H^+$  and the other half-space  $H^-$ . Let  $B = \{+, -\}$ . For  $H \in \mathcal{A}$ , define a map  $\epsilon_H^+ : \mathbf{Ch} \rightarrow B$  by  $\epsilon_H^+(C) = +$  (if  $C \subseteq H^+$ ) and  $\epsilon_H^+(C) = -$  (if  $C \subseteq H^-$ ). Define  $\epsilon_H^- = -\epsilon_H^+$ . Let  $\mathbf{Ch}^m = \mathbf{Ch} \times \mathbf{Ch} \times \cdots \times \mathbf{Ch}$  ( $m$  times). Then the maps  $\epsilon_H^\pm$  induce the maps  $\epsilon_H^\pm : \mathbf{Ch}^m \rightarrow B^m$ . We will study the admissible maps  $\Phi : \mathbf{Ch}^m \rightarrow \mathbf{Ch}$  which are compatible with every  $\epsilon_H^\pm$ . Suppose  $|\mathcal{A}| \geq 3$  and  $m \geq 2$ . Then we will show that  $\mathcal{A}$  is indecomposable if and only if every admissible map is a projection to a component. When  $\mathcal{A}$  is a braid arrangement, which is indecomposable, this result is equivalent to Arrow's impossibility theorem in economics. We also determine the set of admissible maps explicitly for every nonempty real central arrangement.

**Alexander Varchenko** (University of North Carolina)

**Title:** Representation theory and differential equations with polynomial solutions only.

Abstract: Let  $M$  be the tensor product of finite-dimensional representations of the Lie algebra  $gl_N$ . Representation theory allows us to construct a linear differential operator  $D = \sum_{k=0}^N (-1)^k T_k(u) (d/du)^{N-k}$  whose coefficients  $T_k(u) : M \rightarrow M$  are commuting linear operators on  $M$  called the Gaudin transfer matrices. It turns out that the kernel of this differential operator consists of polynomials only and the commutative algebra of its coefficients is related to the algebra of functions on the intersection of suitable Schubert cycles.

**Sergey Yuzvinsky** (University of Oregon)

**Title:** Completely reducible hypersurfaces in a pencil.

Abstract: We study completely reducible fibers of pencils of hypersurfaces on the complex projective space and associated codimension one foliations of this space. Using methods from theory of foliations we obtain certain upper bounds for the number of these fibers as functions only of the dimension of the space. For instance, on every space of dimension greater than 1 this number is less than 6. Since these fibers can be completely characterized by property of their union we obtain upper bounds for the dimensions of resonance varieties of hyperplane arrangements. We obtain also similar bounds for the dimensions of the characteristic varieties of the arrangement complements.

The new results of the talk are from a joint preprint with Jorge V.Pereira