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Title: De-looping embedding spaces.

Abstract: I will survey what is known about the homotopy-type of the space of smooth embeddings of one sphere in another, $Emb(S^j, S^n)$. Denote the space of embeddings of R^j in R^n which agree with a fixed linear embedding outside of a fixed ball by $K_{n,j}$, the 'long' embedding space. One basic result is that $Emb(S^j, S^n)$ fibres over a Stiefel manifold with fibre $K_{n,j}$. Let $K_{n,j}^+$ denote the space of long embeddings of R^j in R^n where each embedding comes with a trivialization of its normal bundle. An observation made four years ago is that $K_{n,j}^+$ admits the action of the operad of little (j+1)-cubes. This makes $K_{n,j}^+$ into a (j+1)-fold loop-space provided $n - j > 2$. Questions related to the structure of $K_{n,j}^+$ as an object over the operad of (j+1)-cubes have motivated much of my recent research. $K_{3,1}^+$ turns out to have the homotopy-type of $K_{3,1} x Z$, and $K_{3,1}$ is a free 2-cubes object over the space of prime long knots. The proof of this is a detailed relationship between the JSJ-decomposition of knot complements and the action of the operad of 2-cubes on $K_{3,1}$. This led to an essentially complete 'computation' of the homotopy-type of $Emb(S^1, S^3)$, which I will outline. Some other developments will be mentioned, such as a generalized Litherland-spinning construction to generate all the Haefliger spheres: $\pi_0 Emb(S^j, S^n)$ for $n - j > 2$, and the computation of the first non-trivial homotopy-groups of $K_{n,j}$ for $2n - 3j - 2 > 0$.