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Title: Complexity and decompositions of PL-manifolds.

Abstract: We define for every PL compact manifold M of dimension n and any integer $0 < k < n$ a non-negative integer $c_k(M)$, called the *k-th complexity* of M, which measures the "minimal complexity" of the k-th stratum of a cellularization of M.

For $n = 2, 3$, c_k is equivalent to well-known invariants such as the Euler characteristic for surfaces, and the Heegaard genus and Matveev complexity for 3-manifolds. Concerning 4-manifolds, c_1 measures the minimum number of 1-handles in a decomposition, and c_2 the minimum number of vertices of a Turaev shadow.

We then focus here on the case $k = 2$ and $n = 4$. The function c_2 leads naturally to a notion of decomposition of any PL 4-manifolds into prime ones, and of prime 4-manifolds into blocks, very similar in nature to the prime and JSJ deomposition of 3-manifolds, with $S^2 \times S^1$ playing the rôle of the torus $S^1 \times S^1$, and (a refined version of) c_2 playing the rôle of Gromov norm. Both decompositions are defined by selecting (among all possible decompositions) only those who preserve the total complexity of the blocks.

Finally, we study the set of 4-manifolds having complexity zero. Pushing further the analogy with 3-manifolds, this set has many features in common with the set of graph manifolds.