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Title: Singular Hecke algebras, Markov traces, and link invariants.

Abstract: A *singular braid* is a braid which admits finitely many transversal double-points. The isotopy classes of singular braids on n strands form a monoid (and not a group) called the *singular braid monoid* and denoted by SB_n. Let $\mathbb{K} = \mathbb{C}(q)$. We define the *singular Hecke algebra* $\mathcal{H}_q(SB_n)$ to be the quotient of the monoid algebra $\mathbb{K}[SB_n]$ by the so-called Hecke relations:

$$\sigma_i^2 = (q-1)\sigma_i + q\,, \quad 1 \le i \le n-1\,,$$

where $\sigma_1, \dots, \sigma_{n-1}$ are the standard generators of the braid group $B_n \subset SB_n$. Following the same approach as Jones for the non-singular Hecke algebras, we define the notion of a *Markov trace* on $\{\mathcal{H}_q(SB_n)\}_{n=1}^{+\infty}$ and show that a Markov trace determines an invariant for singular links. Our main result is that the Markov traces form a graduated vector space $TR = \oplus_{d=0}^{+\infty} TR_d$, where TR_d is of dimension $d+1$. The space TR_0 is spanned by the Oceanu trace, and, for $d \ge 0$, TR_d is the space of traces defined on braids with d singular points. Thanks to this result, we can define a universal Markov trace which gives rise to a universal Jones-type invariant for singular links. This invariant turns to be a Laurent polynomial on 4 variables which can be computed by means of generalized skein relations. (Joint with Loïc Rabenda).