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Title: A new necessary and sufficient condition for the faithfulness of the Burau representation for $B(4)$ (the braid group with 4 strings).

Abstract: It is known that the Burau representation for the braid group $B(n)$ is unfaithful for n greater or equal to 5 (thank to Moody, Long-Patton, Bigelow) and faithful for $n = 3$ (Magnus-Peluso). The remaining case $n = 4$ is unsolved. Having in mind the "unitary" character of the Burau representation, I recently proved that the faithfulness for $n = 4$ is equivalent to the fact that two explicit very "simple" matrices of $SU(3)$ ("simple" meaning that they are stabilization in two different ways of matrices of $SU(2)$) generate a free (non abelian) group. This pair of matrices is a priori much easier to handle than the one in Birman book (theorem 3.19). The question of freeness for the group generated by pairs of matrices has been intensively studied around 1960 for pairs of $SO(3)$ which look very much like the pair I got (see DeGroot, Dekker, ...). Unfortunately their hypotheses are not fulfilled in our case. However I have strong hope to conclude in a near future.