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Title: Chambers of Arrangements of Hyperplanes and Arrow's Impossibility Theorem.

Let $\mathcal{A}$ be a nonempty real central arrangement of hyperplanes and $\mathbf{Ch}$ be the set of chambers of $\mathcal{A}$. Each hyperplane H makes a half-space H^+ and the other half-space H^-. Let $B = \{+, -\}$. For $H \in \mathcal{A}$, define a map $\epsilon_H^+ : \mathbf{Ch} \to B$ by $\epsilon_H^+(C) = +$ (if $C \subseteq H^+$) and $\epsilon_H^+(C) = -$ (if $C \subseteq H^-$). Define $\epsilon_H^- = -\epsilon_H^+$. Let $\mathbf{Ch}^m = \mathbf{Ch} \times \mathbf{Ch} \times \cdots \times \mathbf{Ch}$ (m times). Then the maps $\epsilon_H^\pm$ induce the maps $\epsilon_H^\pm : \mathbf{Ch}^m \to B^m$. We will study the admissible maps $\Phi : \mathbf{Ch}^m \to \mathbf{Ch}$ which are compatible with every $\epsilon_H^\pm$. Suppose $|\mathcal{A}| \geq 3$ and $m \geq 2$. Then we will show that $\mathcal{A}$ is indecomposable if and only if every admissible map is a projection to a component. When $\mathcal{A}$ is a braid arrangement, which is indecomposable, this result is equivalent to Arrow's impossibility theorem in economics. We also determine the set of admissible maps explicitly for every nonempty real central arrangement.