The cohomology of the Milnor fibre of an arrangement with symmetry

The University of Sydney

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NSW 2006
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September 2014, Cortona
This is joint work with Alex Dimca, Nice

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1. Introduction

Let $V = \mathbb{C}^\ell$, $\mathcal{A}$ a hyperplane arrangement in V and G a finite subgroup of $\mathrm{GL}(V)$ such that $G\mathcal{A} = \mathcal{A}$.

To $H \in \mathcal{A}$, corresponds $\ell_H \in V^*$ and we write $Q_0 := \prod_{H\in\mathcal{A}} \ell_H \in \mathbb{C}[V]$, and $d = \deg(Q_0) = |\mathcal{A}|$.

For $g \in G$, $gQ_0 = \lambda_{\mathcal{A}}(g)Q_0$ for $\lambda_{\mathcal{A}}(g) \in \mathbb{C}^\times$, and $\lambda_{\mathcal{A}}$ is a character of G. Write e for $|\lambda_{\mathcal{A}}|$, and note that $Q := Q_0^e$ is G-invariant.

Define the Milnor fibre of $\mathcal{A}$: $F = Q^{-1}(1)$, and the *reduced* Milnor fibre: $F_0 = Q_0^{-1}(1)$; $m = \deg(Q) = de$.

The group $\Gamma := G \times \mu_m$ fixes Q, and hence acts on F. If $\Gamma_0 = \{(g, \xi) \in \Gamma \mid (g, \xi)Q_0 = Q_0\}$, then Γ_0 acts on Q_0.

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The basic problems

Problem A: Determine $P^\Gamma(F,t) := \sum_{i\geq 0} H^i(F,\mathbb{C})t^i$ as an element of $R(\Gamma)[t]$, the Grothendieck ring..

'Determine' might mean: find the character

Problem A': same as A, but with F_0, Γ_0. We'll see soon that problems A,A' are equivalent.

Recall that $H^j(F)$ has two canonical filtrations: the increasing weight filtration W, and the decreasing Hodge filtration F.

These give rise to a mixed Hodge structure on H^j: write

$$H^{p,q}(H^j(F,\mathbb{C})) := \mathrm{Gr}_F^p \mathrm{Gr}_{p+q}^W H^j(F,\mathbb{C})$$

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The second, much harder, problem:

Problem B: Determine the Poincaré-Deligne polynomial $PD^{\Gamma}(F;u,v,t) := \sum_{p,q,j} H^{p,q}H^j(F)u^p v^q t^j$ as an element of $R(\Gamma)[u,v,t]$.

We also have, correspondingly, Problem B' for F_0, Γ_0.

Note that Problem A is the specialisation of Problem B at $u = v = 1$.

We shall meet several other specialisations; a complete solution for Problem B is beyond reach, even in the simplest cases.

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Example

Let $\mathcal{A}$ be the arrangement of type A_ℓ. Then $d = \frac{\ell(\ell+1)}{2}$, $e = 2$ ($\lambda = \varepsilon$) and $m = \ell(\ell+1)$.

$\Gamma = \mathrm{Sym}_{\ell+1} \times \mu_m$ acts on $F : \prod_{i \neq j}(x_i - x_j) = 1$.

This example has motivated much work on this problem–applications in mathematical physics (monopoles-cf. G. Segal, Selby).

Confession: not only is not much known in general about the solution to Problems A and B, but not much is known even about this special and very explicit case, where we have every advantage such as Lie theory.

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Here is what is known:

- $P^G(F/\mu_m, t) = P^\Gamma(F,t)^{\mu_m}$ is known (GL, 1987). The character is given by a product formula analogous to Arn'old's for the Poincaré polynomial. This is a special case of results applying to all unitary reflection groups.

- The above result amounts to the cohomology of the associated hyperplane complement, which is cohomologically pure.

- F is anything but pure; its cohomology has a rich mixed Hodge structure

- $P^{\mu_m}(F/G, t) = P^\Gamma(F,t)^G$ was computed by de Concini-Procesi-Salvetti in 2001.

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- Denham-Lemire (2002) computed the Euler characteristic $P^{\Gamma}(F, -1)$ as a special case of a result about unitary reflection groups.

More about this later

- Settepanella (2004, 2009) computed some low degree examples of $P^{\mu_d}(F_0, t)$ and gave some stability results for this polynomial for the classical groups.
- Like de Concini-Procesi and Salvetti, Settepanella used the Salvetti complex to compute the cohomology.

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Some basic results

Let γ_i denote the character $\zeta \mapsto \zeta^i$ of any group $\mu_r \subset \mathbb{C}^\times$ of roots of unity.

If $(g, \xi) \in \Gamma$, since $Q_0 \in \mathbb{C}[V]$ is homogeneous of degree d, then $(g, \xi)Q_0(v) = \xi^d Q_0(g^{-1}v) = \lambda_{\mathcal{A}}(g)\xi^d Q_0(v)$.

So $\Gamma_0 = \ker(\lambda_{\mathcal{A}} \otimes \gamma_d) \subseteq G \times \mu_m$.

It follows that any representation θ of Γ_0 may be lifted to a representation $\widetilde{\theta}$ of Γ,

and $\mathrm{Ind}_{\Gamma_0}^{\Gamma}(\theta) = \widetilde{\theta}(\sum_{i=0}^{e-1}(\lambda \otimes \zeta_d)^i)$.

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But for all p, q, j, $H^{p,q}H^j(F, \mathbb{C}) \cong \mathrm{Ind}_{\Gamma_0}^{\Gamma}(H^{p,q}H^j(F_0, \mathbb{C}))$.

This is because $F = \amalg_{\zeta \in \mu_e} F_0(\zeta)$, where $F_0(\zeta)$ is given by $Q_0(v) = \zeta$.

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Proposition (Zarelua): Let G act freely on the CW complex X and suppose that $X/G \simeq$ a finite CW complex. Then $\chi^G(X) = P^G(X,-1) = \chi(X/G)\mathrm{Reg}_G$.

Next consider the diagram:

$$\begin{array}{ccc} \widetilde{M} & \xrightarrow{p_1} & M \\ \pi_1 \downarrow & & \downarrow \pi \\ F & \xrightarrow{p} & U = \mathbb{P}(M) \end{array}$$

where M is the hyperplane complement of $\mathcal{A}$ and $\widetilde{M} = \{(v,\xi) \in V \times \mathbb{C}^\times \mid Q(v) = \zeta^m\}$.

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By Zarelua, we have: (i) $\chi^G(F) = \chi(F/G)\mathrm{Reg}_G$ and (ii) $\chi^{\mu_m}(F) = \chi(F/\mu_m)\mathrm{Reg}_{\mu_m} = \chi(U)\mathrm{Reg}_{\mu_m}$.

If G is a unitary reflection group, then $P_M(t) = \prod_{i=1}^{\ell}(1 + m_i^* t)$,

where $m_1^*, \ldots, m_\ell^*$ are the *coexponents* of G.

i.e. the degrees of the generators of $(\mathbb{C}[V] \otimes V)^G$.

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$(g, \xi)v = \xi^{-1}gv = v \iff v \in V(g, \xi)$,
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Hence Γ acts freely on $F^0 := F \setminus \cup_{g \in G, \xi \in \mathbb{C}} V(g, \xi)$. This leads (via Zarelua, applied to a stratification of F by spaces $F(d)^0$ like F^0 for smaller G) to the following result of Denham-Lemire:

If G has invariant degrees $d_1, \dots, d_\ell$ and $d \in \mathbb{Z}_{>0}$, define $\overline{d} := \gcd\{d_i : d|d_i\}$.

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Theorem (Denham, Lemire 2002):
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The following result makes their result into a closed formula.

Theorem (Dimca-L): We have
$\chi(U^0/G) = |Z(G)| \sum_{d \in \mathcal{P}} \mu_{\mathcal{P}}(d) |G(d)|^{-1} \prod_{i \geq 2} (1 - m_i^*(d))$.

We have computed $P^\Gamma(F, t)$ for types A_2, A_3, A_4 and all 2-dimensional groups. We also noticed that in type A_ℓ, we have

$$\dim H^*(F/\mathrm{Sym}_{\ell+1}) = 2\ell + 1.$$

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Relations between monodromy and cohomology degree

We work now with the reduced Milnor fibre F_0.

Since F_0 is an unramified μ_d-covering of U, for any character $\gamma \in \hat{\mu}_d$, we have $H^j(F_0, \mathbb{C})^\gamma \cong H^j(U, L_\gamma)$,

where L_γ is the local system on U, corresponding to γ.

To prove vanishing results for the $H^j(U, L_\gamma)$, we need a compactification of U, which we now describe:

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$\mathcal{L} = \mathcal{L}(\mathcal{A})$ is the lattice of intersections of the hyperplanes in $\mathcal{A}$. Its elements will be called *edges*.

Say that $\mathcal{A}$ is *reducible* if $V = V_1 \oplus V_2$, $V_i \neq 0$, and $\mathcal{A} = \mathcal{A}_1 \amalg \mathcal{A}_2$,

where for each $H \in \mathcal{A}_i$, $H \supseteq V_{i'}$, $(\{i, i'\} = \{1, 2\})$.

For $X \in \mathcal{L}$ we have the arrangement $\mathcal{A}_X := \{H \in \mathcal{A} \mid X \subseteq H\}$. Say that X is *dense* if $\mathcal{A}_X$ is irreducible.

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Define a compactification Z of U along the divisor $N = \cup_{H \in \mathcal{A}} \mathbb{P}(H)$ as follows.

First blow $\mathbb{P}(V)$ up along the 1-dimensional dense edges, then the 2-dimensional ones, etc.

We obtain a resolution $p : Z \longrightarrow \mathbb{P}(V)$, such that $D := p^{-1}(N)$ is a normal crossing divisor D in Z,

with smooth irreducible components D_X, where X runs over the dense edges in $\mathcal{L}$.

Further, p induces an isomorphism : $Z \setminus D \xrightarrow{\sim} U$.

If L_γ ($\gamma \in \hat{\mu}_d$) is the local system (above) on U, then the monodromy of L_γ about the irreducible component D_X is γ^{m_X}, where $m_X = |\mathcal{A}_X|$.

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Define a compactification Z of U along the divisor $N = \cup_{H \in \mathcal{A}} \mathbb{P}(H)$ as follows.

First blow $\mathbb{P}(V)$ up along the 1-dimensional dense edges, then the 2-dimensional ones, etc.

We obtain a resolution $p : Z \longrightarrow \mathbb{P}(V)$, such that $D := p^{-1}(N)$ is a normal crossing divisor D in Z,

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These facts may be used to prove:

Theorem: Suppose that $H^p(F_0,\mathbb{C})^\gamma \neq 0$ for some $\gamma \in \hat{\mu}_d$. Then there is a dense edge $X \in \mathcal{L}$ such that $\mathrm{codim}(X) \leq p+1$ and $|\gamma|$ divides m_X.

Corollary:(Assume $\mathcal{A}$ is essential). If γ is faithful (i.e. $|\gamma| = d$), and $H^p(F_0)^\gamma \neq 0$, then $p = \ell - 1$ (the top degree) and $(\gamma, H^{\ell-1}(F_0))_{\mu_d} = |\chi(U)| (= \prod_{i\geq 2}(m_i^* - 1)$ if $\mathcal{A}$ is a reflection arrangement).

Proof: If $H^p(F_0,\mathbb{C})^\gamma \neq 0$, then by the theorem there is a dense edge $X \in \mathcal{L}$ with $\mathrm{codim}(X) \leq p+1$ and $d|m_X$. But for any $X \in \mathcal{L}$ with $X \neq 0$, $m_X = |\mathcal{A}_X| < |\mathcal{A}| = d$.

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Mixed Hodge structure.

Recall we have

$$H^{p,q}(H^j(F,\mathbb{C}) := \mathrm{Gr}_F^p \mathrm{Gr}_{p+q}^W H^j(F,\mathbb{C}),$$

and the Poincaré-Deligne polynomial:

$PD^\Gamma(F; u, v, t) := \sum_{p,q,j} H^{p,q}H^j(F)u^p v^q t^j.$

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Hodge-Deligne and the spectrum

For an essential arrangement $\mathcal{A} \subset V = \mathbb{C}^\ell$,

the spectrum is defined by

$Sp(\mathcal{A}) := \sum_{\alpha \in \mathbb{Q}} m_\alpha t^\alpha$, where

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where h is the generator of the monodromy.

It has recently been shown by Budur and Saito that $Sp(\mathcal{A})$ depends only on $\mathcal{L}(\mathcal{A})$, not on $\mathcal{A}$.

Proposition Let

$M^{(p)} = (-1)^{\ell-1} \sum_j (-1)^j \mathrm{Gr}_F^p \widetilde{H}^j(F_0, \mathbb{C}) (\in R(\mu_d))$.

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$$Sp(\mathcal{A}) = \left(\sum_{p=0}^{\ell-1} M^{(p)} t^{\ell-1-p}, \sum_{j=1}^{d} \gamma_j t^{\frac{j}{d}} \right)_{\mu_d}.$$

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It follows that the information encoded by the spectrum is precisely the μ_d module structure of the virtual modules $M^{(p)}$.

Corollary: $HD^{\mu_d}(F_0; u, 1)$ depends only on the combinatorics of the arrangement $\mathcal{A}$.

Theorem: Suppose $\mathcal{A}$ is essential in $\mathbb{C}^\ell$. Let $\gamma \in \hat{\mu}_d$ be such that for all dense $X \in \mathcal{L}$, $X \neq 0$, $\gamma^{m_X} \neq 1$. Then $H^p(F_0)^\gamma \oplus H^p(F_0)^{\overline{\gamma}}$ is 0 for $p < \ell - 1$ and is a pure Hodge structure of weight $\ell - 1$ if $p = \ell - 1$.

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The formula in the Proposition also gives complete information about the Hodge structure of $H^1(F)$ in certain cases, e.g. when $\mathcal{A} = A_\ell$.

It is known that in this case, $H^1(F)^{\mu_m}$ is a pure Hodge structure of weight 2, and we have already seen that its cohomology classes are all Tate-of type $(1, 1)$.

Further, $H^1(F) = H^1(F)^{\mu_m} \oplus H^1(F)'$, and $H^1(F)'$ is pure, of weight 1; it therefore contains classes only of type $(1, 0)$ and $(0, 1)$. Moreover, $H^1(F)' = 0$ if $\ell \geq 4$

Since $H^1(F)^{\mu_m} \cong \Theta - 1$ as G-module, where Θ is the permutation action of G on $\mathcal{A}$, we have $\dim H^1(F)^{\mu_m} = \frac{(\ell+2)(\ell-1)}{2}$.

It is easy to compute the spectrum in the cases $\ell = 2, \ell = 3$. Thus the Hodge structure of $H^1(F)$ is completely understood for the braid arrangement.

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An example.

Let $\mathcal{A}$ be the arrangement of type A_4 in $\mathbb{C}^5$.

Then $\Gamma = \mathrm{Sym}_5 \times \mu_{20}$, $\Gamma_0 = \ker(\varepsilon \otimes \gamma_{10})$, and we have the following formula for $P^{\Gamma}(F, t)$.

$P^{\Gamma}(F, t) = (1 \otimes \gamma_0 + \varepsilon \otimes \gamma_{10}) P_0^{\Gamma_0}(F_0, t)$, where

$P_0^{\Gamma_0}(F_0, t) = 1 + [\rho \otimes \gamma_0 + \chi^{(3,2)} \otimes \gamma_0] t +$
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There is still much to be done, but some fascinating hints as to what is happening.

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