Parabolic approximation of a class of hyperbolic initial boundary value problems

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The talk will be based on a joint work with Stefano Bianchini (SISSA, Trieste) and will deal with an initial boundary value problem for a family of mixed hyperbolic-parabolic systems in one space variable:

$$\begin{cases} u^\varepsilon_t + f(u^\varepsilon)_x = \varepsilon \Big(B(u^\varepsilon) u^\varepsilon_x \Big)_x & u^\varepsilon \in \mathbb{R}^N \\ ß(u^\varepsilon(t, 0)) \equiv \bar{g} \\ u^\varepsilon(0, x) \equiv \bar{u}_0. \end{cases} \tag{1}$$

The function ß is a technical tool introduced to handle the case of a singular matrix B. When B is invertible, (1) takes the simpler form

$$\begin{cases} u^\varepsilon_t + f(u^\varepsilon)_x = \varepsilon \Big(B(u^\varepsilon) u^\varepsilon_x \Big)_x & u^\varepsilon \in \mathbb{R}^N \\ u^\varepsilon(t, 0) \equiv \bar{u}_b \\ u^\varepsilon(0, x) \equiv \bar{u}_0. \end{cases} \tag{2}$$

In both (1) and (2) the data $\bar{u}_b$, $\bar{g}$ and $\bar{u}_0$ are constant. I will focus on the analysis of the limit of u^ε as $\varepsilon \to 0^+$. Namely, we will assume that u^ε converges, that the the total variation is sufficiently small and that other technical hypotheses hold. Under these conditions, we will discuss a characterization of the limit.

The problem is non trivial because in general a weak solution of a conservation law

$$u_t + f(u)_x = 0$$

is not unique, so it is interesting to look for a selection principle. Also, in general the limit of (1) and (2) changes if we keep the data $\bar{u}_b$, $\bar{g}$ and $\bar{u}_0$ fixed, but we let the matrix B vary.