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Title: Integrability and quasi-periodic solutions for the Degasperis-Procesi equation

Abstract: The Degasperis-Procesi (DP) equation

$$u_t + c_0 u_x + \gamma u_{xxx} - \alpha^2 u_{xxt} = \left(c_2 (u_x^2 + uu_{xx}) - \frac{2c_3}{\alpha^2} u^2 \right)_x \quad c_0, \alpha, \gamma, c_2, c_3 \in \mathbb{R}$$

provides a mathematical model for nonlinear shallow waters phenomena. In 2002 Degasperis-Holm-Hone proved the integrability of this equation.

Since the DP equation is quasi-linear the presence of dispersive terms depends on the chosen frame. In absence of these terms there are no constants of motion even controlling the H^1 -norm.

We show that, in the dispersive case, there are infinitely many constants of motion which are analytic and control the Sobolev norms in some neighborhoods of the origin.

Moreover, thanks to an analysis of the algebraic structure of these conserved quantities we prove that the (formal) Birkhoff normal form on the circle is action-preserving (integrable) at any order.

This fact is used to provide the first existence result of quasi-periodic solutions for the Degasperis-Procesi equation on the circle.

Based on joint works with R. Feola, S. Pasquali and M. Procesi.