

Workshop “Dispersive equations of Math Physics”  
Department of Mathematics, University of Pisa  
February 6-8, 2020

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## **Titles and abstracts of the talks**

**Paolo Antonelli**  
(GSSI, L'Aquila, Italy)

**Abstract:**

**Jacopo Bellazini**  
(University of Sassari, Italy)

**Abstract:**

**Andrew Comech**  
(Texas A&M University, USA and IITP, Moscow, Russia)

**On global attraction to the manifold of solitary waves**

**Abstract:** The solitary resolution conjecture states that any finite energy solution asymptotically is a superposition of outgoing solitary waves and the dispersive wave. While there is no proof of this general statement, we prove the following two results:

- (1) Any global finite energy solution to a generic nonlinear Klein-Gordon equation which is assumed to have compact spectrum (the support of the Fourier transform in time) converges to the set of solitary waves;
- (2) Any global finite energy solution to the Klein-Gordon equation with the nonlinearity on the compact part of a hypersurface indeed has compact spectrum, and consequently converges to the set of solitary waves.

The proof is based on the Titchmarsh convolution theorem and its version for the partial convolution.

**Piero D'Ancona**  
(Sapienza University of Rome, Italy)

**Global Strichartz estimates for an inhomogeneous Maxwell system**

**Abstract:** We study a model of electromagnetic wave propagation in a continuous medium which is asymptotically homogeneous, namely a Maxwell system with linear, isotropic, inhomogeneous material laws. The main results are: a sharp resolvent estimate for the time harmonic case, obtained by combining Carleman and Morawetz estimates; a global smoothing estimate for the time dependent equation; and a sharp Strichartz estimate in the same range of indices as for propagation in vacuum. The result is a joint work with R.Schnaubelt (Karlsruhe)

**Giulio Del Corso**  
(GSSI, L'Aquila, Italy)

**Abstract:**

**Michele Dolce**  
(GSSI, L'Aquila, Italy)

**Separation of time-scales in a drift-diffusion equation with radial advection on  $2 - d$**

**Abstract:** Consider a passive scalar that is advected by a radial power law velocity field and it is diffused with a diffusivity coefficient  $k < 1$ . Physically and numerically it is known that the passive scalar is mixed on a time-scale much faster with respect to the purely diffusive one (think of milk and coffee). We identify the time-scale (sharp up to a log correction) at which mixing happens along the streamlines of the background flow. In this case, the scalar becomes firstly axisymmetric and afterward only diffusion across streamlines it is possible. The time-scale is proportional to  $k^{-c}$  where  $c < 1$  depends on the strength of the background shear. The proof of the result is based on an adaptation of the hypocoercivity scheme and yield to exponential convergence towards a radially symmetric configuration. This is a joint work with Michele Coti Zelati.

**Donatella Donatelli**  
(Department of Information Engineering, Computer Science and Mathematics, University of L'Aquila, Italy)

**Dispersive behaviour in the scale limit analysis in fluid dynamics modelling**

**Abstract:** The so called fluid dynamical models are used in different contest such as physics, astrophysics, geophysics and environmental sciences in order to model the various arising phenomena. Eliminating unwanted or unimportant modes of motion, and keeping the essential balances between flow fields, allow us to better focus and to deeper understand the problem. Scaling and asymptotic analysis play an important role in this approach. By scaling the equations, meaning by choosing appropriately the system of the reference units, the parameters determining the behavior of the system become explicit. Asymptotic analysis provides a useful tool in the situations when certain of these parameters called characteristic numbers vanish or become infinite. On the other hand it turns out that a large part of models used in fluid dynamics rely on a formal asymptotic analysis of more complex system. Therefore it is important, from a mathematical point of view to perform a rigorous scaling limit analysis in order to understand the behaviour of the various fluid dynamical models according to the different domains and initial conditions. The common feature of this kind of limit in the ill prepared data framework is, the presence of high frequency time oscillations along the so called acoustic waves. Those waves

are supported by the gradient part of the velocity field, which, as consequence becomes infinite.

In this scenario it becomes extremely important to understand under which conditions the behaviour of these waves can be controlled and their presence is harmless. In this talk by means of different models we will show in which cases those waves have a dispersive behaviour and how dispersion may help in order to recover the necessary compactness of the velocity field.

## Bibliography

- [1] D. Donatelli and P. Marcati, Quasineutral limit, dispersion and oscillations for Korteweg type fluids, *SIAM J. Math. Anal.* 47, no. 3, (2015), 2265-2282
- [2] D. Donatelli and E. Feireisl, An anelastic approximation arising in astrophysics, *Math. Ann.* 369, (2017), 1573–1597.
- [3] D. Donatelli, Scale analysis for an atmosphere flow, [arXiv:1811.09808](https://arxiv.org/abs/1811.09808), [math.AP].

## Tianxiang Gou (Chinese Academy of Sciences, China)

### Standing waves to nonlinear Chern-Simons-Schrödinger equation with prescribed $L^2$ -norm

**Abstract:** In this talk, we are mainly concerned with the existence, uniqueness and dynamical behaviors of standing waves to nonlinear Chern-Simons-Schrödinger equation with prescribed  $L^2$ -norm. This is a joint work with Prof. Zhitao Zhang.

## Ning-An Lai (Lishui University, China)

### Lifespan estimate for semilinear Tricomi equations

**Abstract:** In this talk, I will introduce some results about the lifespan estimate for semilinear generalized Tricomi equations. This is a joint work with N.M. Schiavone.

## Yuan Li (Lanzhou University, China and University of Pisa, Italy)

### The Blowup solutions for the mass critical fractional Schrödinger equation

**Abstract:** We consider the fractional Schrödinger equation with mass critical in one dimension

$$i\psi_t = D^s \psi - |\psi|^{2s} \psi,$$

where  $D$  denotes the first order fractional derivative and  $1 < s < 2$ . In this paper, we prove the existence of finite-time blowup solutions with minimal mass  $\|\psi_0\|_2 = \|Q\|_2$ , where  $Q$  is the ground state solution of the equation  $D^s Q + Q = Q^{2s} Q$ . The analysis for the construction of minimal blow up elements introduced by J. Krieger, E. Lenzmann and P. Raphaël.

**Sandra Lucente**

**(Dipartimento Interateneo di Fisica, University of Bari, Italy)**

**Fujita regularity exponent for scale invariant damping semilinear wave equation**

**Abstract:** We consider the Cauchy problem

$$v_{tt} - \Delta v + \frac{\mu}{1+t} v_t + \frac{\mu}{2} \left( \frac{\mu}{2} - 1 \right) \frac{v}{(1+t)^2} = |v|^p$$

where  $v = v(t, x)$  with  $x \in \mathbb{R}^n$ ,  $\mu > 0$ ,  $p > 1$ . This equation had been studied by many authors in the last decade. Some known results describe suitable conditions on  $\mu$ ,  $p$ ,  $n$  which implies non-existence of weak solutions or blow-up dynamic for  $L^p$  norms of the local solution. Other results establish global existence results provided suitable relations on  $\mu$  and  $p$ . In this case Strauss and Fujita critical exponents for wave equations and heat equations comes into play. If we consider radial solution in weighted  $L^p$  spaces another parameter appears. Indeed the weight of the solution space inherits the decaying of the initial data. Let us denote by  $k > 0$  the order of a polynomial decay of the initial data. Fixed  $\mu$  and  $n$ , the known global existence results are valid in a particular zone of a  $p, k$  plane. In this talk, after summarizing the known results, we will describe the existence zone as a supergraph of a curve of Fujita-type. Finally we will present a new result of blowup type below such curve.

**Pierangelo Marcati**

**(GSSI, L'Aquila, Italy)**

**Abstract:**

**Elena Nikolova**

**(Technical University, Sofia, Bulgaria)**

**Abstract:**

**Raffaele Scandone**  
(GSSI, L'Aquila, Italy)

**Global well-posedness for the non-linear Maxwell-Schrödinger system**

**Abstract:** In this talk I will discuss the global well-posedness and the growth of Sobolev norms for the Maxwell-Schrödinger system with an additional pure-power non-linearity. The main ingredients are suitable a-priori bounds, based on a frequency-localized version of the Strichartz estimates, combined with the technique of modified energies. As an important byproduct of the a-priori bounds, it follows that the Lorentz force is well defined for ordinary solutions in the energy space.

I will also discuss the existence of global, finite energy, weak solutions to a magnetic quantum hydrodynamic system, which is formally related to the non-linear Maxwell-Schrödinger system by means of the Madelung transform.

Based on joint work with P. Antonelli and P. Marcati.

**Nico Michele Schiavone**  
(Sapienza University of Rome, Italy)

**Heat-like and wave-like behaviour of the lifespan estimates for wave equations with scale-invariant damping and mass**

**Abstract:** In this talk we consider blow-up results and lifespan estimates in the subcritical-case for wave equations with scale-invariant damping and mass and power-nonlinearity. The focus of this talk is to show how the competition between the 'heat-like' and 'wave-like' behaviours appears not only in the definition of the critical exponent, but also in the lifespan estimates, which transit from one behaviour to the other in dependence of the damping and mass coefficients and in dependence of the exponent in the nonlinearity. Moreover, we see that the lifespan estimates can be different also under certain conditions on the initial data. This work is in collaboration with Ning-An Lai (Lishui University) and Hiroyuki Takamura (Tohoku University).

**Boris Shakarov**  
(GSSI, L'Aquila, Italy)

**Abstract:**

**Koichi Taniguchi**  
(Nagoya University, Japan)

**Global dynamics for semilinear heat equations in energy spaces associated with self-adjoint operators**

**Abstract:** In this talk, we consider the global behavior of solutions to the initial-boundary

value problems for semilinear heat equations with lower energy initial data than the mountain pass level in energy spaces associated with self-adjoint operators satisfying Gaussian upper bounds. This talk is based on the joint work with M. Ikeda (RIKEN / Keio University).

**Mirko Tarulli**  
**(Technical University, Sofia, Bulgaria)**

**Decay and Scattering in energy space for the solution of generalised Hartree equation**

**Abstract:** We prove decay with respect some Lebesgue norms for a class of Schrödinger equations with non-local nonlinearities by showing new Morawetz inequalities and estimates. As a straightforward product we obtain large-data scattering in the energy space for the solutions to the defocusing generalized Hartree equations with mass-energy inter-critical nonlinearities in any space dimensions.

**Nicola Visciglia**  
**(University of Pisa, Italy)**

**Abstract:**