# Qualitative Properties of Dispersive PDEs

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*Georgiev, Michelangeli, Scandone organisers*

Valeria Banica (*Laboratoire Jacques-Louis Lions, Sorbonne Université Paris*)

**Growth of the energy for the binormal flow and the Schrödinger map**

*Summary* : In this talk I shall consider the binormal flow equation, which is a model for the dynamics of vortex filaments in Euler equations. Geometrically it is a flow of curves in three dimensions, explicitly connected to the 1-D Schrödinger map with values on the 2-D sphere, and to the 1-D cubic Schrödinger equation. The result that I will present is that, although these equations are completely integrable, we show the existence of an unbounded growth of the energy density. The density is represented by the amplitude of the high frequencies of the derivative of the tangent vectors of the curves, thus giving information of the oscillation at small scales. In the setting of vortex filaments the variation of the tangent vectors is related to the one of the direction of the vorticity, that according to the Constantin-Fefferman-Majda criterion plays a relevant role in the possible development of singularities for Euler equations. This is a joint work with Luis Vega.

Jacopo Bellazzini (*University of Pisa*)

**Dynamical collapse of cylindrical symmetric Dipolar Bose-Einstein condensates**

*Summary* : We study the formation of singularities for cylindrical symmetric solutions to the Gross-Pitaevskii equation describing a dipolar Bose-Einstein condensate. We prove that solutions arising from initial data with energy below the energy of the Ground State and that do not scatter collapse in finite time. Joint work with Luigi Forcella.

Federico Cacciafesta (*University of Padua*)

**Dispersive estimates for the Dirac-Coulomb equation**

*Summary* : The Dirac-Coulomb equation represents a relevant model in relativistic quantum mechanics, as well as a challenging one from the point of view of dispersive PDEs: indeed, the Coulomb potential is critical with respect to the scaling of the (massless) Dirac operator, and therefore proving dispersive estimates is a complicated task. We will present some families of dispersive estimates (namely local smoothing and Strichartz estimates with loss of angular derivatives) that we recently obtained, trying to highlight the main differences and difficulties with respect to its non-relativistic counterpart, the Schroedinger equation.The talk is based on works in collaboration with E. Séré (Paris CEREMADE) and J. Zhang. (Beijing Institute of Technology).

Scipio Cuccagna (*University of Trieste*)
**Refined profiles and selection of small standing waves**

*Summary* :  We discuss the notion of Refined Profile, which simplifies considerably the proof of the selection of standing waves in the context of small energies. This is joint work with Masaya Maeda.

Piero D'Ancona (*University of Rome La Sapienza*)
**Localisation of eigenvalues for non-selfadjoint operators**

*Summary* : We give a group of results on the localization of eigenvalues, or absence thereof, for Dirac operators in arbitrary dimension, perturbed by non hermitian potentials of sufficiently small size in critical norms. In particular, we extend to higher dimensions the 2014 one dimensional result due to Cuenin, Laptev and Tretter. The main tools we use are an abstract Birman-Schwinger principle combined with suitable sharp resolvent estimates. The results are contained in joint papers with L. Fanelli, D. Krejcirik and N. Schiavone.

Damiano Foschi (*University of Ferrara*)
**Minimal smoothness nonlinear interactions for the well-posedness of nonlinear dispersive equations**

*Summary* : We investigate the relation between regularity of initial data and regularity of the function which describes the nonlinear interaction in order to have local well-posedness for semilinear dispersive equations with power-like nonlinear terms.

Renato Lucà (*BCAM Bilbao*)
**Transport of Gaussian measures with exponential cut-off for Hamiltonian PDEs**

*Summary* : We study the evolution of suitable weighted Gaussian measures for some periodic Hamiltonian PDEs such as the quintic nonlinear Schrödinger equation and the Benjamin-Bona-Mahony equation. The new ingredient is the introduction of an exponential cut-off on a high order Sobolev norm that allows to simplify and extend earlier results. The results have been obtained in collaboration with Giuseppe Genovese and Nikolay Tzvetkov.

Sandra Lucente (*University of Bari*)
**Linear and Nonlinear interaction for wave equations with time variable coefficients.**

*Summary* : Many papers deal with the nonlinear wave equation with variable coefficients. The interaction between the operator and the nonlinear term is evident in terms of critical exponents for global existence vs blow-up. When the nonlinear term depends on the derivatives of the solution, the situation is even more delicate. Indeed,

yet in the constant coefficients case, the null conditions strongly relate the symbol of the linear operator with the form of admissible nonlinear terms which leads to global existence. For some peculiar time-dependent coefficients operator one can find a change of variable that takes such operator in the form of wave equation with covariant time derivative. Also here, it becomes evident the interaction between nonlinear and the linear terms. In particular the parameters that appears in this covariant derivative will enter in some sufficient conditions for the global existence results. In the present work we explore the case of nonlinear terms which depends also on the second covariant derivative. As a byproduct, this enables us to generalize the null conditions for a class of variable coefficients operators.

Diego Noja (*University of Milan Bicocca*)
**The nonlinear Schrödinger equation with isolated singularities**

*Summary* : A nonlinear Schrödinger (NLS) equation with a point interaction and power nonlinearity in dimension two and three is introduced and well-posedness is considered. Behind the autonomous interest of the problem, this is a model of the evolution of so called singular solutions that are well known in the analysis of semilinear elliptic equations. We show that the Cauchy problem enjoys local existence and uniqueness of strong solutions, and that the solutions depend continuously from initial data. In dimension two well posedness holds for any power nonlinearity and global existence is proved for powers below the cubic. In dimension three local and global well posedness are restricted to low powers. (Work in collaboration with Claudio Cacciapuoti and Domenico Finco.)

Tohru Ozawa (*University of Waseda*)
**Zakharov System in Two Space Dimensions**

*Summary* : We study the Cauchy problem for the Zakharov system in a two dimentional domain. Under natural assumptions on the data, we prove the existence and uniqueness of global solutions in $H^2 \oplus H^1$. The method of the construction of global solutions depends on the proof that solutions of some regularized system form a bounded sequence in $H^2 \oplus H^1$ and a Cauchy sequence in $H^1 \oplus L^2$. The method of proof is independent of the compactness argument and Brezis-Gallouet inequality. This is my recent joint work with Kenta Tomioka, Waseda University.

Michela Procesi (*University of Rome III*)
**Linearization for infinite dimensional Hamiltonian systems**

*Summary* : I shall consider classes of analytic infinite dimensional Hamiltonian dynamical systems in the neighborhood of an elliptic fixed point. I shall show that if the linear frequencies satisfy a Diophantine-like condition and if the Hamiltonian is formally symplectically conjugated to its quadratic part, then it is also analytically symplectically conjugated to it. Of course what is an analytic symplectic change of variables depends strongly on the choice of the phase space. Here we work on periodic functions with Gevrey regularity. Based on joint work with L. Stolovitch

Nikolay Tzvetkov (*University of Cergy-Pontoise*)
**Transport of Gaussian measures under the flow of Hamiltonian PDE's**

*Summary* : The transport of Gaussian measures under transformations in an infinite dimensional space is a delicate issue as shown by the classical Cameron-Martin theorem (1944). We will show that a natural class of Gaussian measures, invariant under the flow of linear Hamiltonian PDE's, remain quasi-invariant under nonlinear (Hamiltonian) perturbations. We will also identify the Radon-Nikodym derivative by using some hidden cancellations. We will make an attempt to keep the presentation at a non technical level by probably sacrificing a little the rigor.

Nicola Visciglia (*University of Pisa*)
**Remarks on the scattering theory for $L^2$ subcritical NLS**

*Summary* : We review a classical paper by Yajima and Tsustsumi in weighted spaces. In particolare we up-grade the scattering from $L^2$ to $H^1$.