$$\langle \phi_k | \phi_{k'} \rangle = \langle \phi_k | \int_{-L/2}^{+L/2} dx\, |x\rangle\langle x| \phi_{k'} \rangle$$

$$\Rightarrow \left(\frac{2\pi}{L} n + k_0\right)\frac{L}{2} = \frac{\pi}{2}(2\ell - 1),\ \ell = 1, 2, \dots \Rightarrow k_0 = -\frac{\pi}{L}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

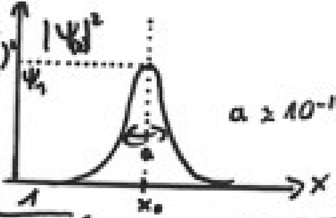
$$\langle \phi_k | \phi_{k'} \rangle = \int_{-L/2}^{+L/2} dx\, \phi_k^*(x) \cdot \phi_{k'}(x)$$

$$\psi_n(x) = \sqrt{\frac{2}{L}} \cos\left[\frac{\pi}{L}(2n-1)x\right];\ \varphi_a - \varphi_b = \pi;\ \psi_n(x) = \sqrt{\frac{2}{L}} \sin\left[\frac{2\pi}{L} n x\right]$$

$$\langle \phi_k | \phi_{k'} \rangle = \frac{1}{L}\int_{-L/2}^{+L/2} dx\, e^{-ikx} e^{ik'x} \stackrel{!}{=} 0;\ k \neq k'$$

$$\hat{H}\psi_{ns}(x) = -\frac{\hbar^2}{2m}\partial_x^2 \psi_{ns}(x) = \frac{\hbar^2}{2m}\left(\frac{\pi}{L}[2n-1]\right)^2 \psi_{ns}(x)$$

$$E_{ns} = \frac{\hbar^2}{2m}\frac{\pi^2}{L^2}(2n-1)^2,\ n = 1, 2, \dots;\ \hat{H}\psi_{na}(x) = \frac{\hbar^2}{2m}\left(\frac{2\pi}{L}\right.$$



$$|\psi(x)|^2 = |\psi_0|^2 e^{-\frac{(x-x_0)^2}{2a^2}}$$

$$a \approx 10^{-10} m$$

$$\int_{-\infty}^{+\infty} dx\, e^{-Ax^2} = \sqrt{\frac{\pi}{A}}$$

$$A = \frac{1}{2a^2} \Rightarrow |\psi_0| = \frac{1}{(2\pi a^2)^{1/4}}$$

$$\hat{H}\psi_a = -\frac{\hbar^2}{2m}\partial_x^2 \psi_a(x) = \frac{\hbar^2}{2m}\frac{1}{2a^2}\psi_a(x) - \frac{\hbar^2}{2m}\frac{1}{4a^4}(x-x_0)^2\psi_a(x)$$

$$= -\frac{\hbar^2}{2m}\left(-\frac{1}{2a^2} + \left(\frac{1}{2a^2}(x-x_0)\right)^2\right) e^{-\frac{(x-x_0)^2}{4a^2}} \psi;\ V(x) = \frac{\hbar^2}{2m}\frac{1}{4a^4}(x-x_0)^2$$

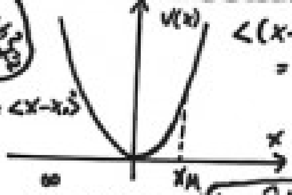
$$\hat{H} \to \hat{H} = -\frac{\hbar^2}{2m}\partial_x^2 + V(\hat{x});\ \hat{H}\psi_a = \frac{\hbar^2}{2m}\frac{1}{2a^2}\psi_a = E_a \psi_a$$

$$V(x) = \frac{1}{2}m\omega^2(x-x_0)^2 \to m\omega^2 = \frac{\hbar^2}{m 4a^4} \Rightarrow \boxed{\omega = \frac{\hbar}{2ma}}$$

$$E_a = \frac{\hbar^2}{2m}\frac{1}{2a}$$

$$[\hat{p}, \hat{x}] = \frac{\hbar}{i};\ \hat{p} = \frac{\hbar}{i}\partial_x\ /\ \hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2$$

$$a^2\hat{p}^2 + b^2\hat{x}^2$$



$$\langle (x-x_0)^2 \rangle = \langle \psi_a | (x-x_0)^2 | \psi_a \rangle$$

$$1.\ a^2 + b^2 = (a+ib)(a-ib);\ a, b \in \mathbb{R};\ 2.\ (a\hat{p}\ ib\hat{x})(a\hat{p} - ib\hat{x}),\ a, b \in \mathbb{R}$$

$$= a^2\hat{p}^2 + iba\hat{x}\hat{p} - iab\hat{p}\hat{x} + b^2\hat{x}^2 = a^2\hat{p}^2 + b^2\hat{x}^2 - ba\hbar$$

$$a^2 = \langle x - x_0 \rangle$$

$$\hat{H} = (a\hat{p} + ib\hat{x})(a\hat{p} - ib\hat{x}) + ba\hbar;\ a^2 = \frac{1}{2m};\ b^2 = \frac{1}{2}m\omega^2$$

$$\int_{-\infty}^{\infty} dx\, |x\rangle\langle x| = \mathbb{1}$$

$$\psi_a^*(x)\langle k|$$

$$\text{Df}:\ C^+ = \frac{1}{\sqrt{\hbar\omega}}(a\hat{p} + ib\hat{x});\ C^- = \frac{1}{\sqrt{\hbar\omega}}(a\hat{p} - ib\hat{x}) \Rightarrow \hat{H} = \hbar\omega c^+ c + \frac{1}{2}\hbar\omega$$

$$\hat{x}|x\rangle = x|x\rangle$$

$$= \int_{-\infty}^{\infty} dx\, \psi_n^*(x)(x-x_0)^2 \psi_a(x)$$

$$\left\{\begin{pmatrix} w & -\bar{t} \\ t & \bar{w} \end{pmatrix} \middle| w, t \in \mathbb{C}\right\}\ \{\pm 1\};\ SU(2) \cong S^3\quad A \to \omega \bar{A} \omega^{-1}$$