Titles abstracts

September 28, 2022

1 Riccardo Adami

Two-dimensional systems with nonlinear point interactions

Nonlinear point interactions were introduced two decades ago in order to describe localized phenomena like trapping, beatings, concentration of wave packets and so on. Seminal studies focused on one and three-dimensional systems, leaving untouched the more technical two-dimensional case up to two years ago. We review the results and comment on the specific features exhibited by this case, together with open problems. This is a joint project with Filippo Boni, Raffaele Carlone, Michele Correggi, and Lorenzo Tentarelli.

2 Ivan Beshastnyi

Self-adjoint extensions for the curvature Laplacian on Grushin manifolds

I will discuss some results from an ongoing project with H. Quan (University of Washington), in which we study the curvature Laplacian on curved high-dimensional analogues of the α-Grushin planes. We are able to prove that all self-adjoint properties of the Laplacian are encoded in a second order polynomial, which can be easily read off from the equation itself. In particular, we can determine whether the operator is essentially self-adjoint by computing its discriminant and construct all of its self-adjoint extensions from its roots.

3 Filippo Boni

NLS ground states with singularities

We investigate the existence of ground states at fixed mass of the L^2 - subcritical NLSE with a point interaction. First, the problem is considered in dimension two and three. We prove that ground states exist for every value of the mass and, up to a multiplication by a phase factor, they are positive,

radially symmetric, decreasing along the radial direction and present a singularity where the interaction is placed. In order to obtain qualitative features of the ground states, we refine a classical result on rearrangements and move to equivalent variational formulations of the problem. Then, we present some future developments, among them the possibility to generalize similar models on hybrid structures. This is a joint work with R. Adami, R. Carlone and L. Tentarelli.

4 Ugo Boscain

Geometric confinement of the curvature laplacian on almost Riemannian manifolds.

Two-dimensional almost-Riemannian structures of step 2 are natural generalizations of the Grushin plane. They are generalized Riemannian structures for which the vectors of a local orthonormal frame can become parallel. Under the 2-step assumption the singular set Z, where the structure is not Riemannian, is a 1D embedded submanifold. While approaching the singular set, all Riemannian quantities diverge. A remarkable property of these structures is that the geodesics can cross the singular set without singularities, but the heat and the solution of the Schrödinger equation (with the Laplace-Beltrami operator Δ cannot. This is due to the fact that (under a natural compactness hypothesis), the Laplace-Beltrami operator is essentially self-adjoint on a connected component of the manifold without the singular set. In the literature such counterintuitive phenomenon is called geometric confinement. For the heat equation an intuitive explanation of this fact can be given in terms of random walk. For the Schrödinger equation an intuitive explanation is more subtle since the evolution of a quantum particle on a manifold can be done in several non-equivalent way. In this talk I will describe the evolution (and the confinement) of a quantum particle described by the curvature Laplacian $-\Delta + cK$ (here K is the Gaussian curvature and c¿0 a constant) which originates in coordinate free quantization procedures (as for instance in path-integral or covariant Weyl quantization).

Joint work with Eugenio Pozzoli and Ivan Beschastnyi

5 Luca Fanelli

About Schrödinger and Dirac operators with scaling critical potentials

Abstract: Lower-order perturbations of the free Hamiltonians usually appear in Quantum Mechanics, ad models describing the interaction of a free particle with an external field. In some cases, the perturbation lies at the same level as the free Hamiltonian, and the resulting conflict can generate interesting phenomena. We will introduce the Inverse Square and Coulomb potentials as toy models, and describe the main features of the complete Hamiltonians from the

point of view of Fourier Analysis, Spectral Theory, and dispersive evolutions. Some recent results in the setting of the Heisenberg Group will be also presented.

6 Valentina Franceschi

Pointed sub-Laplacians in three dimensions and Hardy inequalities

he aim of this seminar is to present some recent results on the essential self-adjointness of pointed sub-Laplacians in three dimensions. We show that, unlike the Euclidean case, pointed sub-Laplacians (associated with smooth measures) are essentially self-adjoint in dimension 3. To this purpose, we focus on the case of the 3D Heisenberg sub-Laplacian, and we show its essential self-adjointness by exploiting non-commutative Fourier transform techniques. We then generalize the result to a class of 3D sub-Riemannian manifolds. In connection with the main result, we present a discussion on Hardy inequalities in the Heisenberg group: contrary to the Euclidean case, a radial Hardy inequality, i.e., a Hardy inequality taking into account only the directional derivative w.r.t. the sub-Riemannian distance, does not hold in this context for any dimension. This underlines again a difference with respect to the Euclidean case, where essential self-adjointness of pointed laplacians can be derived from Hardy inequalities. Based on a joint work with R. Adami (Politecnico di Torino), U. Boscain (CNRS & UPMC, Sorbonne Université), and D. Prandi (CNRS & Centralesupélec)

7 Matteo Gallone

The Laplace-Beltrami operator on the Grushin Cylinder

Abstract: When a quantum particle is constrained on an orientable Riemannian manifold, one challenging problem that arises naturally is the question of the so-called 'geometric quantum confinement'. This is the possibility that a particle whose initial wavefunction is supported inside some portion of space may remain confined in such a region for all times when evolving according to the unitary group generated by the free Hamiltonian. This occurrence is related to the presence of singularities in the metric and to the (essential) self-adjointness of the Laplace-Beltrami. The prototypical example of space exhibiting this phenomenon is the 'Grushin-like cylinder' that is, roughly speaking, a cylinder with metric $ds^2 = dx^2 + 1/|x|^(2\alpha)dy^2$. In this talk I will consider this manifold and I will present the classification of a physically interesting sub-family of self-adjoint realisations of the Laplace-Beltrami operator in the regime where it is not essentially self-adjoint. I will discuss the advantages of the usage of Krein-Vishik-Birman self-adjoint extension theory.

This is based on a series of joint works with A. Michelangeli and E. Pozzoli.

8 Marilena Ligabó

Boundary conditions, product formulae and classical limit

In this seminar some connection between (classical and quantum) boundary condition and product formulae will be illustrated. In particular, using the Trotter product formula, a composition law for quantum boundary conditions will be introduced. Furthermore, the link between the occurrence of boundary conditions and frequent measurements on a quantum system will be discussed. This phenomenon is due to the existence of a one-parameter unitary group obtained as the limit of the so called Zeno product formula (Zeno dynamics). Finally, it will be shown how the classical limit of a particular Zeno dynamics induces a change of topology in phase space.

9 Diego Noja

The NLS equation with a point interaction in two and three dimensions.

The NLS equation with a power nonlinearity and point interaction (a "delta potential") is well studied in dimension one but only recently the two and three dimensional case have been considered. In the talk, the well posedness of the Cauchy problem will be treated and a discussion of some asymptotic properties will be given, in particular as regards the blow-up for strong nonlinearities and the absence of scattering for long range nonlinearity. Work in collaboration with Claudio Cacciapuoti and Domenico Finco

10 Mario Rastrelli

On the squared Laplacian perturbed with singular potential

It will be given an explicit characterization of the domain of the Laplacian with inverse squared potential. The methods used will be improved to describe the domain of the operator squared in the radial case and in the general one.

11 Raffaele Scandone

NLS WITH POINT INTERACTIONS: RESULTS AND PERSPECTIVES

In this talk I will review some recent advances in the study of non-linear Schrödinger equations with delta-like potentials. After introducing a suitable functional framework, the notion of sub-critical non-linearity, and a class of dispersive-type estimates, I will discuss some results concerning the global well-posedness and the existence (and symmetries) of standing waves. The last part of the talk will be devoted to open questions and future research perspectives.

12 Ivana Vojnović

Generalised solutions to non–linear Schrödinger equations with singularities

We consider Schrödinger equations of Hartree and cubic type in three spatial dimensions and its approximations of singular, point-like perturbations.

As approximants to the Hartree equation, we analyze the equation of the form

$$i\partial_t u_\varepsilon \;=\; -\Delta u_\varepsilon + V_\varepsilon u_\varepsilon + (w * |u_\varepsilon|^2)u_\varepsilon \,, \tag{1}$$

for $\varepsilon \in (0,1]$. Here V_ε is a real-valued potential and is meant to represent a singular, delta-like profile centered at $x = 0$.

We assume that

$$V_\varepsilon(x) \;:=\; \frac{1}{\varepsilon^\sigma} V\Big(\frac{x}{\varepsilon}\Big)\,, \tag{2}$$

for a given measurable function $V : \mathbb{R}^3 \to \mathbb{R}$ and a given $\sigma \geqslant 0$.

The corresponding nets of approximate solutions represent generalised solutions for the singular-perturbed Schrödinger equation. The behaviour of such nets is investigated for $\sigma \in [0,3]$.

We also study a generalised solution in the Colombeau algebra $\mathcal{G}_{C^1,H^2}$ for cubic and Hartree equation with delta potential. In the case of the Hartree equation with delta potential compatibility between the Colombeau solution and the solution of the classical Hartree equation is established.

Results are based on joint work with Nevena Dugandžija and Alessandro Michelangeli.

13 Jens Wirth

Laplace and sub-Laplace operators on homogeneous groups and non-commutative Fourier transforms

During the talk I will introduce basic notions related to non-commutative Fourier transforms on different groups and their relation Laplace and sub-Laplace operators. This can be seen as a starting point for a (pseudo-differential) calculus and used to study further problems in appropriate Sobolev spaces.