### **Markov Chains**

Steve Kirkland, National University of Ireland, Maynooth Michael Neumann, University of Connecticut, USA

### On the mean first passage matrix of a simple random walk on a tree

R.B. BAPAT, Indian Statistical Institute New Delhi, India rbb@isid.ac.in

Mon 17:35, Room B

We consider a simple random walk on a tree. Exact expressions are obtained for the expectation and the variance of the first passage time, thereby recovering the known result that these are integers. A relationship of the mean first passage matrix with the distance matrix is established and used to derive a formula for the inverse of the mean first passage matrix.

# Probabilistic Approach to Perron Root, the Group Inverse, and Applications

IDDO BEN ARI, University of Connecticut, Storrs, USA

Mon 17:10, Room B

A probabilistic approach to the study of the Perron root of irreducible nonnegative matrices is presented. This is then applied to reestablish and improve some known results in the field. The analysis focuses on perturbative theory for the Perron root the group inverse of a generator of a continuous time Markov chain, and their relations.

#### Restricted additive Schwarz methods for computing the stationary vector of large Markov chains

MICHELE BENZI, Emory University, Atlanta, GA, USA benzi@mathcs.emory.edu

Wed 12:15, Room B

The Restricted Additive Schwarz (RAS) algorithm is a domain decomposition method that has proved very effective in solving large sparse systems of linear equations arising from the discretization of partial differential equations on parallel computers. In this talk we extend the RAS algorithm to the problem of computing the steady-state (stationary) vector of Markov chains with large state spaces. We prove convergence of the stationary iterative method, and we address computational issues such as partitioning, the amount of overlap, inexact subdomain solves, the construction of two-level schemes bases on "coarse grid" corrections, and Krylov subspace acceleration. The results of numerical experiments on matrices arising from real applications in Markov modelling will be presented.

Joint work with Verena Kuhlemann (Emory University)

#### Coupling and Mixing in Markov Chains

JEFFREY J HUNTER, Auckland University of Technology & Masey University, New Zealand

Wed 11:25, Room B

Following a discussion of the concepts of mixing and coupling in Markov chains, expressions for the expected times to mixing and coupling are developed. The two-state cases and three-state cases are examined in detail and some results

for the bounds on the expected values are given. The key results are given in Hunter, J.J.: "Coupling and mixing times in a Markov chain", Linear Algebra and its Applications, 430, 2607-2621, (2009), and Hunter, J.J.: "Bounds on Expected Coupling Times in a Markov Chain", (pp271-294), "Statistical Inference, Econometric Analysis and Matrix Algebra. Festschrift in Honour of Gtz Trenkler", Bernhard Schipp and Walter Kraemer (Editors), Physica-Verlag Heidelberg (2009)

#### To be Announced

Ali Jadbabaie, University of Sydney, Australia

Wed 11:50, Room B

## Non-negative matrix products and John Hajnal (1924-2008)

EUGENE SENETA, School of Mathematics and Statistics, FO7, University of Sydney, Australia

Mon 16:45, Room B

Hajnal (Proc. Cambridge Philos. Soc., 54 (1958), 233-246) gave the first proof of a necessary and sufficient condition for weak ergodicity of a sequence of nxn stochastic matrices (non-negative matrices with row sums one). Intrinsically, he used the Markov-Dobrushin coefficient of ergodicity, whose sub-unit value indicates contractivity for a stochastic matrix.

He also introduced the key idea of a scrambling matrix. Later (Math. Proc. Cambridge Phil. Soc., 79(1976)521-530), motivated by work in demography on inhomogeneous products of certain kinds of non-stochastic non-negative matrices, he developed a weak ergodicity theory for general non-negative matrix products, using Birkhoff's contraction ratio. Both papers were enormously influential on subsequent developments.

This tribute to Hajnal outlines his biography, motivation and methodology, and briefly synthesizes the history of inhomogeneous products pre- and post- 1957, including the work of Doeblin and Sarymsakov.

# The Inverse Mean First Passage Matrix Problem And The Inverse M-Matrix Problem

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Wed 11:00, Room B

The inverse mean first passage time problem is given an  $n \times n$  positive matrix M, then when does there exist an n state discrete time homogeneous ergodic Markov chain, whose mean first passage matrix is M. The inverse M-matrix problem is given a nonnegative matrix A, then when is A an inverse of an M-matrix. In this talk, results concerning these two problems are discussed.

Joint work with M. Neumann, University of Connecticut