Si consideri

$$f(x,y) = \begin{cases} x - y \arctan\left(\frac{x}{y}\right) & y \neq 0 \\ x & y = 0 \end{cases}$$

i) stud. la cont. di f su $\mathbb{R}^2$

ii) studi. la diff. di f in $\mathbb{R}^2$

iii) calcolare il max e min di f su $\{ |x|^2 + |y|^2 \leq 1 \} = D$

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i) contin. in $(\bar{x}, \bar{y})$ per $\bar{y} \neq 0$

$$f(\xi, h) = \xi - h \arctan\left(\frac{\xi}{h}\right) \xrightarrow{(\xi,h)\to(\bar{x},0)} \bar{x}$$

$\Rightarrow \quad f \in C^0(\mathbb{R}^2)$

ii) differenz. in $(\bar{x}, \bar{y})$, $\bar{y} \neq 0$

consid. $(\bar{x}, 0)$ con $\bar{x} \neq 0$

$$\frac{f(\bar{x}, h) - f(\bar{x}, 0)}{h} = -\arctan\left(\frac{\bar{x}}{h}\right)$$

non ha lim per $h \to 0$

quindi f non è diff. in $(\bar{x}, 0)$
per $\bar{x} \neq 0$

nel caso $(\bar{x}, \bar{y}) = (0, 0)$

$$\frac{f(h,0)}{h} \equiv 1 \implies \exists\, \partial_x f(0,0) = 1$$

$$\frac{f(0,h)}{h} \equiv 0 \implies \exists\, \partial_y f(0,0) = 0$$

consid. poi

$$\frac{f(h,k) - h}{(h^2+k^2)^{1/2}} = \frac{-k \arctan\left(\frac{h}{k}\right)}{\sqrt{h^2+k^2}}$$

$$= -\frac{k}{\sqrt{h^2+k^2}} \arctan\left(\frac{h}{k}\right)$$

$(h,k) = t(1, \lambda) \implies$

$$= -\frac{\lambda}{\sqrt{1+\lambda^2}} (\operatorname{sgn} t) \arctan\left(\frac{1}{\lambda}\right) \qquad \lambda \neq 0$$

$\implies f$ non è diff. in $(0,0)$

pertanto f non è diff. su $\{(x,0) \mid x \in \mathbb{R}\}$

iii) $f(x,0) = x$

$$f_x = 1 - \cancel{y}\frac{1}{\left(1+\frac{x^2}{y^2}\right)}\frac{1}{\cancel{y}} = 1 - \frac{y^2}{x^2+y^2}$$

$$= \frac{x^2}{y^2+x^2} \quad \text{per } y \neq 0$$

$$f_y = -\arctan\left(\frac{x}{y}\right) - y\frac{1}{1+\left(\frac{x}{y}\right)^2}\left(-\frac{x}{y^2}\right)$$

$$= -\arctan\left(\frac{x}{y}\right) + \frac{xy}{x^2+y^2} \quad \text{per } y \neq 0$$

quindi

$y \neq 0 \quad \nabla f = 0 \iff y \neq 0 \quad x = 0$

Pertanto i pti di min e max globali stanno in $\{(x,y): x^2+y^2=1\} \cup \{(x,0): |x| \leq 1\}$

Scegliamo la parametrizzazione $\gamma(t) = (\sin t, \cos t)$, $-\frac{\pi}{2} < t \leq \frac{3}{2}\pi$, della circonferenza $\{(x,y): x^2+y^2=1\}$

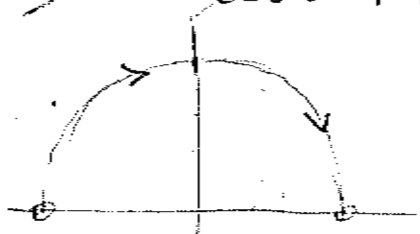
$$f(\gamma(t)) = \sin t - (\cos t)\arctan(\operatorname{tg} t)$$

per $t \in \left]-\frac{\pi}{2}, \frac{\pi}{2}\right[\implies$

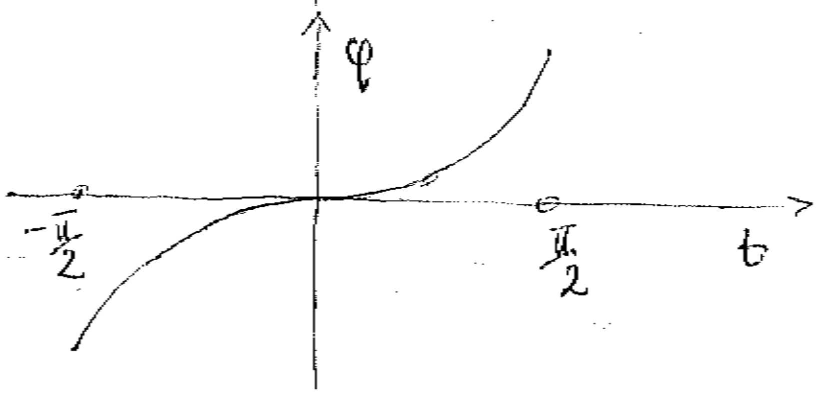
$$f(\gamma(t)) = \sin t - t\cos t$$

$$\varphi(t) = \sin t - t\cos t \qquad -\frac{\pi}{2} < t < \frac{\pi}{2}$$

$$\varphi'(t) = \cancel{\cos t} - \cancel{\cos t} + t\sin t = t\sin t \geq 0$$



$$(\sin t, \cos t)\Big|_{\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)}$$



$$f(x,y) = f(x,-y) \implies$$

$$\varphi\left(\frac{\pi}{2}\right) = f\left(\gamma\left(\frac{\pi}{2}\right)\right) = f(1,0) = \max_C f$$

ove $C = \{(x,y) : x^2+y^2=1\}$

$$\varphi\left(-\frac{\pi}{2}\right) = f\left(\gamma\left(-\frac{\pi}{2}\right)\right) = f(-1,0) = \min_C f$$

$$\implies \max_D f = f(1,0) = 1$$

$$\min_D f = f(-1,0) = -1$$