APPROXIMATE EXTENSION OF PARTIAL
\(\varepsilon\)-CHARACTERS OF ABELIAN GROUPS TO
CHARACTERS WITH APPLICATION TO INTEGRAL
POINT LATTICES

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Let \(G\) be an abelian group, \(S \subseteq G\) be a finite set, and \(\mathbb{T}\) denote the multiplicative group of complex units with the invariant arc metric \(|\arg(a/b)|\).

We will show that for a mapping \(f : S \rightarrow \mathbb{T}\) to be \(\varepsilon\)-close on \(S\) to a character \(\varphi : G \rightarrow \mathbb{T}\) it is enough that \(f\) be extendable to a mapping \(\bar{f} : (S \cup \{1\} \cup S^{-1})^n \rightarrow \mathbb{T}\), where \(n\) is big enough and \(\bar{f}\) violates the homomorphy condition at most up to an arbitrary \(\delta < \min(\varepsilon, \pi/2)\). Moreover, \(n\) can be chosen uniformly, independently of \(G\) and both \(f\) and \(\bar{f}\), depending just on \(\delta, \varepsilon\) and the number of elements of \(S\).

The proof is non-constructive, using a special case of Gordon’s nonstandard version of Pontryagin-van Kampen duality [1], [2] or, alternatively, the ultraproduct construction and the classical Pontryagin-van Kampen duality, hence yielding no estimate on the actual size of \(n\).

As one of the applications we show that, for a vector \(u \in \mathbb{R}^q\) to be \(\varepsilon\)-close to some vector from the dual (polar, reciprocal) lattice \(H^*\) of a full rank integral point lattice \(H \subseteq \mathbb{Z}^q\), it is enough for the scalar product \(ux\) to be \(\delta\)-close (with \(\delta < 1/3\)) to an integer for all vectors \(x \in H\) satisfying \(\sum_i |x_i| \leq n\), where \(n\) depends on \(\delta, \varepsilon\) and \(q\) only.

References


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