

**PROBLEMS IN ADDITIVE NUMBER THEORY WITH
AN ULTRAFILTER FLAVOR**

MELVYN B. NATHANSON

The set A of nonnegative integers is called a *basis of order h* if every nonnegative integer can be represented as the sum of h not necessarily distinct elements of A . The positive real number α is called an *additive eigenvalue of order h* if there is a basis $A = \{a_i\}_{i=0}^{\infty}$ of order h such that $a_i \sim \alpha i^h$. The set of all additive eigenvalues of order h is called the *spectrum of order h* , and denoted $\mathcal{N}(h)$. Cassels showed that additive eigenvalues exist for all $h \geq 2$. It is proved that $0 < \eta(h) = \sup \mathcal{N}(h) \leq h!/\Gamma^h(1 + 1/h)$, and that the spectrum $\mathcal{N}(h)$ is an interval of the form $(0, \eta(h))$ or $(0, \eta(h)]$. A preprint will be appear on arXiv.org.

DEPARTMENT OF MATHEMATICS, LEHMAN COLLEGE AND THE GRADUATE CENTER, CITY UNIVERSITY OF NEW YORK, NEW YORK U.S.A.

E-mail address: `melvyn.nathanson@lehman.cuny.edu`
`nathansn@alpha.lehman.cuny.edu`