PIT for Weakly Dicomplemented Lattices

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Outline

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Motivation

Boolean aglebras vs Powerset algebras

- X a set. $(\mathcal{P}(X), \cap, \cup, ^c, X, \emptyset)$ powerset algebra.
- $(B, \wedge, \vee, ', 0, 1)$ Boolean algebra.
- $SB :=$ all ultrafilters on B
- Endow SB with a topology having $(N_a)_{a \in B}$ as basis, where $N_a := \{U \in SB \mid a \in U\}$.
- $CSB :=$ clopen subsets of SB.
- $B \cong CSB \leq \mathcal{P}(SB)$. (Stone)

Problem: abstract vs concrete

Weakly dicomplemeted lattices vs concept algebras
What is the equational theory of concept algebras?

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Definition and examples

Definition

A **weakly dicomplemented lattice** is an algebra $(L; \wedge, \vee, ^{\triangle}, ^{\triangledown}, 0, 1)$ of type $(2,2,1,1,0,0)$, where $(L; \wedge, \vee, 0, 1)$ is a bounded lattice and the equations $(1) \ldots (3')$ hold.

(1) $x^{\triangle\triangle} \leq x$, **(1')** $x^{\triangledown\triangledown} \geq x$,

(2) $x \leq y \implies x^{\triangle} \geq y^{\triangle}$, **(2')** $x \leq y \implies x^{\triangledown} \geq y^{\triangledown}$,

(3) $(x \wedge y) \vee (x \wedge y^{\triangle}) = x$, **(3')** $(x \vee y) \wedge (x \vee y^{\triangledown}) = x$.

$^{\triangle}$ is called a **weak complementation**, $^{\triangledown}$ a **dual weak complementation** and $(^{\triangle}, ^{\triangledown})$ a **weak dicomplementation**.

Definition and examples

- Boolean algebra: duplicate the complementation. $(B, \wedge, \vee, ', 0, 1) \rightsquigarrow (B, \wedge, \vee, ', ', 0, 1)$
- pseudocomplemented (*) and dual pseudocomplemeted ($^+$) distributive lattices. $(L, \wedge, \vee, ^+, ^*, 0, 1)$.
- Bounded lattice:

$$x \neq 1 \implies x^{\triangle} := 1 \quad \text{and} \quad x \neq 0 \implies x^{\triangledown} := 0.$$

- L finite lattice. $G \supseteq J(L)$ and $N \supseteq M(L)$ where $J(L)$ is the set of join irreducible elements of L and $M(L)$ its set of meet irreducible elements. For $x \in L$. Define

$$x^{\triangle} := \bigvee\{g \in G \mid g \not\leq x\} \text{ and } x^{\triangledown} := \bigwedge\{n \in N \mid n \not\geq x\}$$

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Contexts and concepts

- **Formal context** :=(G, M, I) with $I \subseteq G \times M$.
  G :≡ set of **objects** and M :≡ set of **attributes**.
- Derivation. $A \subseteq G$ and $B \subseteq M$.
  $A' := \{m \in M \mid \forall g \in A \quad gIm\}$
  $B' := \{g \in G \mid \forall m \in B \quad gIm\}$.
- **Formal concept** := a pair (A, B) with $A' = B$ and $B' = A$.
  A :≡ **extent** of (A, B) and B :≡ **intent** of (A, B).
  $\mathfrak{B}(G, M, I) :=$ set of all concepts of (G, M, I).
- **Concept hierarchy**
  $(A, B) \leq (C, D) :\iff A \subseteq C \quad (\iff D \subseteq B)$.
- $\underline{\mathfrak{B}}(G, M, I) := (\mathfrak{B}(G, M, I), \leq)$

The Basic Theorem on Concept Lattices

Theorem

$\underline{\mathfrak{B}}(G, M, I)$ is a complete lattice in which infimum and supremum are given by:

$$\bigwedge_{t \in T} (A_t, B_t) = \left(\bigcap_{t \in T} A_t, \left(\bigcup_{t \in T} B_t \right)'' \right)$$

$$\bigvee_{t \in T} (A_t, B_t) = \left(\left(\bigcup_{t \in T} A_t \right)'', \bigcap_{t \in T} B_t \right).$$

$\underline{\mathfrak{B}}(G, M, I)$ is called the **concept lattice** of the context (G, M, I).

The Basic Theorem on Concept Lattices

Theorem

A complete lattice L is isomorphic to a concept lattice $\underline{\mathfrak{B}}(G, M, I)$ iff there are mappings $\tilde{\gamma} : G \to L$ and $\tilde{\mu} : M \to L$ such that $\tilde{\gamma}(G)$ is supremum-dense in L, $\tilde{\mu}(M)$ is infimum-dense in L and for all $g \in G$ and $m \in M$

$$gIm \iff \tilde{\gamma}(g) \leq \tilde{\mu}(m).$$

In particular $L \cong \underline{\mathfrak{B}}(L, L, \leq)$.

Some special contexts

Finite lattices $L \cong \underline{\mathfrak{B}}(J(L), M(L), \leq)$.

Powerset algebras $\underline{\mathfrak{B}}(X, X, \neq) \cong \mathcal{P}X$.

Distributive lattices $\underline{\mathfrak{B}}(P, P, \not\geq) \cong \mathcal{O}(P, \leq)$.

Boolean Concept Logic

conjunction via meet

disjunction via join

negation ?Hmmm!

Weak Negation $(A, B)^{\triangle} := ((G \setminus A)'', (G \setminus A)')$

Weak opposition $(A, B)^{\triangledown} := ((M \setminus B)', (M \setminus B)'')$.

$x \vee x^{\triangle} = 1$ but $x \wedge x^{\triangle}$ can be different of 0;

Definition

The algebra $\mathfrak{A}(\mathbb{K}) := (\mathfrak{B}(\mathbb{K}), \wedge, \vee, ^{\triangle}, ^{\triangledown}, 0, 1)$ is called the **concept algebra** of $\mathbb{K}$.

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Concept algebras: some equations

1. $x^{\triangle} \leq y \iff y^{\triangle} \leq x$,
2. $(x \wedge y)^{\triangle\triangle} \leq x^{\triangle\triangle} \wedge y^{\triangle\triangle}$,
3. $x^{\triangledown\triangledown\triangledown} = x^{\triangledown} \leq x^{\triangle} = x^{\triangle\triangle\triangle}$.

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2. $(x \vee y)^{\triangledown\triangledown} \geq x^{\triangledown\triangledown} \vee y^{\triangledown\triangledown}$.
3. $x^{\triangle\triangledown} \leq x^{\triangle\triangle} \leq x \leq x^{\triangledown\triangledown} \leq x^{\triangledown\triangle}$.

- $x \mapsto x^{\triangle\triangle}$ is an interior operator on L.
- $x \mapsto x^{\triangledown\triangledown}$ is a closure operator on L.

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Find an axiomatization of concept algebras.

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Find an axiomatization of concept algebras.

Representation problem

strong representation
: Describe weakly dicomplemented lattices that are isomorphic to the concept algebras.

equational axiomatization
: Find a set of equations that generate the equational theory of concept algebras.

concrete embedding
: Given a weakly dicomplemented lattice L, is there a context $\mathbb{K}(L)$ such that L can be embedded into the concept algebra of $\mathbb{K}(L)$?

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Prime Ideal Theorem

Definition

A primary filter is a proper filter F of L such that for all $x \in L$, $x \in F$ or $x^{\triangle} \in F$. A primary ideal is a proper ideal I of L such that for all $x \in L$, $x \in F$ or $x^{\triangledown} \in I$.

Theorem (PIT)

Let F a filter and I an ideal of L such that $F \cap I = \emptyset$. Then there is a primary filter G containing F such that $G \cap I = \emptyset$.

Corollary (separation)

If $x \not\leq y$ there is a primary filter G with $x \in G$ and $y \notin G$.

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Canonical context

- $\mathfrak{F}_{pr}(L) :=$ set of primary filters of L
- $\mathfrak{I}_{pr}(L) :=$ set of primary ideals of L
- $\mathbb{K}(L) := (\mathfrak{F}_{pr}(L), \mathfrak{I}_{pr}(L), \Delta)$ with $F \Delta I :\iff F \cap I \neq \emptyset$.
- $\mathfrak{F}_x := \{F \in \mathfrak{F}_{pr}(L) \mid x \in F\}$ and $\mathfrak{I}_x := \{I \in \mathfrak{I}_{pr}(L) \mid x \in I\}$.

Theorem

The mapping

$$\begin{array}{rccc} \varphi : & L & \to & \underline{\mathfrak{B}}(\mathbb{K}(L)) \\ & x & \mapsto & (\mathfrak{F}_x, \mathfrak{I}_x) \end{array}$$

is a lattice embedding.

- $\mathfrak{F}_x' = \mathfrak{I}_x$ and $\mathfrak{I}_x' = \mathfrak{F}_x$.
- $\mathfrak{F}_{x \wedge y} = \mathfrak{F}_x \cap \mathfrak{F}_y$ and $\mathfrak{I}_{x \vee y} = \mathfrak{I}_x \cap \mathfrak{I}_y$.

Dreamlike embedding

Wdl embedding

Is φ a weakly dicomplemented lattice embedding?

What about the weak operations?

- $\mathfrak{I}_{x^\triangle} \subseteq (\mathfrak{F}_{pr}(L) \setminus \mathfrak{F}_x)'$
- $\mathfrak{F}_{x^\triangledown} \subseteq (\mathfrak{I}_{pr}(L) \setminus \mathfrak{I}_x)'$

Thus $\varphi(x^\triangledown) \leq \varphi(x)^\triangledown \leq \varphi(x)^\triangle \leq \varphi(x^\triangle)$.

Where is the problem?

Let I be a primary ideal such that $I \not\ni x^\triangle$. If $x \notin I$ but $x^\triangle \in Ideal(I \cup \{x \wedge x^\triangle\})$, **is there a primary filter** F **such that** $x \notin F$ **and** $F \cap I = \emptyset$?

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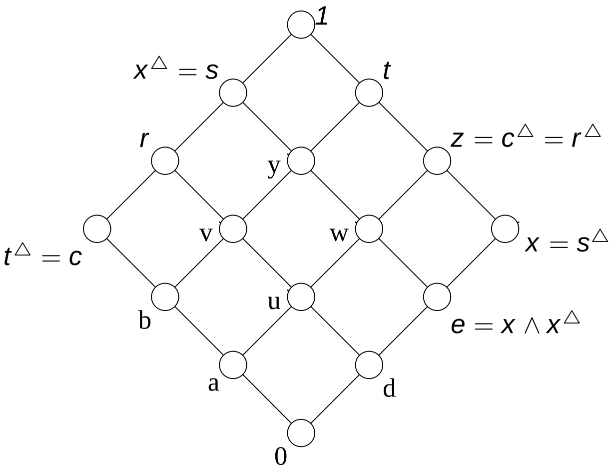
Illustration



Conjecture: strong separation

Let I be a primary ideal such that $I \not\ni x^{\triangle}$. Assume that $I \not\ni x$ and $x^{\triangle} \in Ideal(I \cup \{x \wedge x^{\triangle}\})$. Then there is a primary filter $F \not\ni x$ such that $F \cap I = \emptyset$.

L is a Boolean algebra

- φ is an embedding.
- $\mathfrak{A}(\mathbb{K}^{\triangle}_{\triangledown}(L))$ is a complete and atomic Boolean algebra.
- $\mathfrak{A}(\mathbb{K}^{\triangle}_{\triangledown}(L))$ is isomorphic to $\mathcal{P}(\mathfrak{F}_{pr}(L))$.
- i.e. New proof of: “every Boolean algebra is a field of sets”

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L is a finite and distributive lattice: solved

But the proof uses combinatorial arguments and is based on a different approach.

L is a finite lattice: open

- (primary) filters are principal and generated by ($\vee$-primary) elements: $\{a \in L \mid a \leq x \text{ or } a \leq x^{\triangle}\ \forall x \in L\}$.

  (primary) ideals are principal and generated by ($\wedge$-primary) elements: $\{a \in L \mid a \geq x \text{ or } a \geq x^{\triangledown}\ \forall x \in L\}$.
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- Impact of the properties of L on $\mathfrak{A}(\mathbb{K}(L))$.
- Topological representations
- Duality

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