GRUPPI E RAPPRESENTAZIONI
2023/24 - II SEMESTRE

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Introduction

This course aims to be an introductory course in representation theory and noncommutative algebra, with a focus on representations of the symmetric group and their combinatorics.

The first part of the course will be an introduction to group representations, a rich and powerful framework that provides a bridge between abstract algebra and the study of symmetries in various contexts. We aim to: understand the basic definitions and properties of representations, characters, and their relation to group actions; explore the decomposition theory of representations, including Maschke's theorem, and gain proficiency in manipulating direct sums and tensor products of representations; investigate the structure and classification of irreducible representations, uncovering the connection to the character theory of finite groups.

The second part of the course will focus on the symmetric group, where a more highly specialized machinery is developed to handle this case. Combinatorial methods play a significant role in the representation theory of the symmetric group: Young tableaux, combinatorial objects that encode permutation patterns, help understanding the irreducible representations using Specht modules. We will also rephrase all these results in terms of the algebra of symmetric functions, which is both one of the objects with the richest structure of all mathematics and an invaluable tool to better manipulate our machinery.

If times permits, we will dip into some other topic, such as the representation theory of general linear groups, which is deeply interwined with most of the previous topics and provides a deeper understanding of the interplay between linear algebra, group theory, and the symmetries inherent in linear transformations.

Practical information

The course will last 48 hours, and it will take place in the second semester. Throughout the course, the students will have the possibility to submit the solutions to some exercises that will be assigned in class: if they do so, that will be counted as part of the exam, the rest of which will consist in a seminar to be presented within the summer exam session; if they do not, they will have to take an oral exam instead, which will include questions on both theory and exercises. The prerequisites for this course are basic linear algebra and basic theory of finite groups. For any further question, please contact michele.dadderio@unipi.it or alessandro.iraci@unipi.it.