GEOMETRIA DIFFERENZIALE COMPLESSA

GREGORY PEARLSTEIN, SPRING 2024

1. Introduction

In simplest terms, a complex manifold is just a differentiable manifold upon which one has a notion of what it means for a smooth function to be holomorphic. One of the striking features of the theory of complex manifolds is that a given differentiable manifold can have many inequivalent complex structures. One often sees this already in classical complex analysis in the form "Two annuli are conformally equivalent iff the ratio of their radii are the same".

This course will develop the theory of complex manifolds and their deformation theory following the classic book *Complex Manifolds* by James Morrow and Kunihiko Kodaira. The basic topics for the course will be:

- Complex manifolds (definitions and examples)
- Sheaves and Cohomology (only the basic theory)
- Geometry of Complex Manifolds (hermitian and Kähler metrics,...)
- Hodge theory

For Hodge theory, we will follow the book *Period Mappings and Period Domains* by Chris Peters, Stefan Müller-Stach, James Carlson.

2. Highlights

We will discuss the period matrix of a Riemann surface and the Torelli theorem. We will also discuss the Hodge decomposition theorem, the Lefschetz (1,1)-theorem and the Hodge conjecture.

3. Practical Information

The course will last 42 hours and take place during the second semester. The course will be taught in English. The course grade will be based on a short paper/presentation by the student. The prerequisites for the course are a basic knowledge of complex analysis (holomorphic functions, Laurent series, residue theorem) and differential geometry (manifolds).