Metodi di Approssimazione

- MSc course (Laurea magistrale); 42 hours, 6 credits.
- 2nd semester (Spring).

What is this course about? Not approximation theory, mostly!

Selected topics in advanced linear algebra, close to (some) practical applications and modern research themes.

Themes

- Methods to compute matrix functions;
- Methods to solve some specific matrix equations;
- Applications to control theory.

Specialized course on advanced topics, not a 'generalist' course on numerical computing.

Movie trailer 1: matrix functions

Given a scalar function $f : U \subseteq \mathbb{C} \to \mathbb{C}$, can we extend it so that it is defined for $A \in \mathbb{C}^{n\times n}$?
You have already seen $\exp(A) = I + A + \frac{A^2}{2} + \dots$

With the natural definition, unexpected derivatives of f appear:

$$f\left(\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}\right) = \begin{bmatrix} f(0) & f'(0) & \frac{1}{2}f''(0) \\ 0 & f(0) & f'(0) \\ 0 & 0 & f(0) \end{bmatrix}.$$

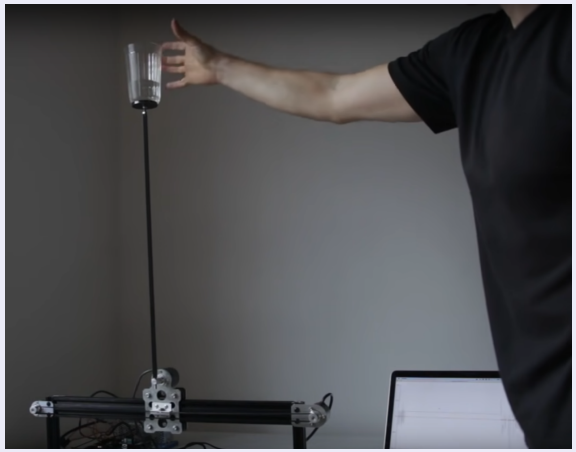
We shall see techniques to compute them involving:

- matrix decompositions;
- some approximation theory: replace f with a 'nearby' polynomial or rational function;
- Iterations, e.g., $X_{k+1} = \frac{1}{2}(X_k + X_k^{-1})$;

Movie trailer 2: matrix equations and control

Control theory

Study of dynamical systems with an 'input' to keep them in an (unstable) equilibrium:



Solving the linear case involves linear algebra and matrix equations, e.g.: find X that solves

$$XCX - AX + XD - B = 0,$$

where all matrices are $n \times n$ square. How?

Info

Prereqs

- Numerical analysis
- Scientific computing

Synergy with other courses from the same area, e.g., numerical methods for Markov chains, numerical methods for ODEs.

Course format

- Frontal lectures with Matlab examples.
- Tablet notes + lecture notes available.

Studying

Books

- Higham *Functions of Matrices*.
- Datta, *Numerical Methods for Linear Control Theory*.

+ lecture notes.

Exam

Presentation on a research paper: theory + implementing numerical examples.