

# Algebraic Number Theory 2 (TAN2)

A.A. 2023/24 - First semester

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## Introduction

The course illustrates the basic results in the theory of local fields, with a particular emphasis on the  $p$ -adic fields, thus providing a natural complement to the theory of global fields (mainly number fields) presented in the Algebraic Number Theory 1 (TAN1) course. The interplay between the local and global results is crucial in all areas of number theory: global objects often inherit the properties of their local counterparts (which are usually easier to study) and the cases in which that is not possible (obstructions to the local-global principle) also provide an interesting area of research.

## Main Features

As a first introduction to local fields, the course shows their construction via completion at non-archimedean absolute values, this already presents some interesting phenomena due to the associated topology.

A large part of the course will be devoted to the extensions of local fields: we study the main properties and characterizations of unramified, tamely ramified and wildly ramified extensions, and with an in-depth study of the ramification filtration and of the group of units.

In this course we shall lay the foundations for local class field theory (whose introduction may or may not be part of the course depending on time and on the interest of the audience), which is in turn a starting point for global class field theory, a powerful tool providing a description of abelian extensions of global fields.

## Practical Information

This is a 42 hours course (6 CFU), and takes place in the first semester.

There will be an oral examination based on exercises and questions about the content of the course.

A few prerequisites are needed, namely a basic knowledge of Arithmetic, Topology and Galois Theory, together with the basic results of the Algebraic Number Theory 1 course.

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