Algebra Superiore A

087AA MAT-L/WMA

A.A. 2023/24 - Second semester

Teacher: Enrico Sbarra

This is an advanced course in Commutative Algebra, which builds on the courses of Algebra 2 and Istituzioni di Algebra. Keeping in mind Cohen's Structure Theorem, we will further develop the theory of finitely generated modules over commutative noetherian rings, including graded rings and modules, Hilbert functions, minimal free resolutions and Cohen-Macaulay rings. We are going to discuss a large variety of classical and modern theorems, together with some open problems and conjectures.

Main Features

After reviewing some preliminary facts for noetherian rings and modules, such as dimension theory, we are going to introduce graded rings and modules, and recall some basic definitions and constructions in the graded case. Next, we study Hilbert functions in detail, as they provide one of the most useful tool to study graded objects. As a main application, we determine properties of coordinate rings of projective varieties. We also show some concrete examples arising from other fields of Mathematics, such as Combinatorics and Algebraic Topology, also with the help of Gröbner bases.

In the second part of the course, we use classic results of homological algebra and the properties of derived functors to define and study Cohen-Macaulay rings, a large class of extremal rings which are endowed with many interesting properties.

Practical Information

- This 42 hours course (6 CFU) will be held in the second semester, and is intended for students of the Master programs in Mathematics.

- The prerequisites consist of the courses of Algebra 2 and Istituzioni di Algebra.

- The final exam consists of a 50-minute blackboard seminar and the solution of some exercises handed in one week before.

- All the topics treated in this course and the related exercises can be found in the notes which will be handed out during the course.

- For further information please visit the website of the course on the ECTS catalogue of the University of Pisa. Please refer to the dedicated section of this homepage for previous editions of the course, exams' rules, older exams, the student opinion questionnaires, *etcetera*.