

A.A. 2023/24 - Second semester

Teachers: Andrea Bandini - Enrico Sbarra

This course provides a first introduction to the fundamental concepts of Commutative Algebra, including rings, ideals, modules, tensor product, localization, Noetherian rings, and primary decomposition. These topics appear in various areas of Mathematics and are crucial tools for students who intend to pursue further studies in Homological Algebra, Algebraic Geometry, Algebraic Number Theory, *etcetera*.

## Main Features

In this course we will introduce and study the theory of commutative unitary rings and finitely generated modules defined over them, together with many theoretical and abstract results, new concepts and techniques.

We aim to illustrate the fundamental theoretical concepts via some algorithms, and concrete and computable exercises. Our main examples are going to be multivariate polynomials rings over a field and modules over a principal ideal domain (PID). The main tools we use to compute such examples are Gröbner bases, which have natural applications in the study of varieties and of polynomial systems of equations, and the Smith canonical form of matrices, by means of which we describe the structure of finitely generated modules over a PID.

## **Practical Information**

- This 60 hours course (6 CFU) will be held in the second semester, and is intended for students of both the Bachelor and Master programs in Mathematics.
- Few prerequisites are needed, namely a basic knowledge of Arithmetic, Abelian Groups, and Linear Algebra.
- The final exam consists of a written and an oral test.
- All the topics treated in this course and the related exercises can be found in this book.
- For further information please visit the website of the course on the ECTS catalogue of the University of Pisa. Please refer to the dedicated section of this homepage for previous editions of the course, exams' rules, older exams, the student opinion questionnaires, *etcetera*.