111AA Mathematical Physics
(Fisica Matematica)
Second semester 2023-24
6 CFU, 42 hours

Instructor:
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Information on the course and the timetable can be found at the webpage
`https://pagine.dm.unipi.it/bonanno/fismat-2324.html`

1 Overview

The aim of the course is to give an introduction to the advanced theory of discrete dynamical systems. The course "Sistemi dinamici" provides an introduction to the basic notions of the theory and to the topological properties of one-dimensional discrete systems. In this course we begin to look at examples in more dimensions and to understand the new phenomena that may appear. In particular, we look at examples of discrete systems that arise as discretisation of flows defined by Hamiltonian systems, hence these examples are defined in n-dimensional spaces, with $n \geq 2$.

Another natural characteristic of physical model is the dependence on one or more parameters. In the theory of Hamiltonian systems it is by now classical to look at integrable systems in which adding a term with a "small" parameter the integrability breaks and chaotic phenomena appear. The same happens to the discretised system. In this course we consider the *Standard map* [1], a two-dimensional discrete dynamical systems on the cylinder which is the prototypical example of a one-parameter depending system exhibiting chaotic phenomena. To understand these phenomena we start studying the classical theory of circle homeomorphisms and the Poincaré-Birkhoff theory for maps on the cylinder. Then we consider the modern approaches that include the Mather's variational method.

2 Course outline

- Reducing continuous-time systems to discrete-time systems: the Poincaré map. Area-preserving Poincaré maps.
- Basic notions of discrete dynamical systems in more dimensions. Smale horseshoe map and topological chaos. Twist maps.
- From integrable to chaotic systems: phenomenology of small perturbations of integrable maps.
- Circle homeomorphisms: rotation number and its properties for one-parameter families of maps; conjugacy to a rotation.
- Twist maps of the cylinder: existence of periodic orbits; existence and non-existence of rotational invariant circles (KAM and converse-KAM theory); Aubry-Mather theory and topological chaos.

[1] `https://en.wikipedia.org/wiki/Standard_map`

3 Bibliography

- J.D. Meiss, "Symplectic maps, variational principles, and transport", Reviews of Modern Physics, vol. **64**, (1992), pag. 795–848
- A. Katok, B. Hasselblatt, "Introduction to the modern theory of dynamical systems", Cambridge University Press, 1995
- J.K. Moser, E.J. Zehnder, "Notes on dynamical systems", Americal Mathematical Society, 2005
- S. Wiggins, "Introduction to applied nonlinear dynamical systems and chaos", Springer, 1990

4 Assessment

The exam is oral. The student which has attended the course may decide to give a seminar on an advanced topic chosen with the instructor. A list of topics is provided during the course. The seminar is followed by a short discussion on the main results of the course.

5 Prerequisites

Basic notions of dynamical systems.