Transcendental Methods in Complex Algebraic Geometry
(Geometria Algebrica B)

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Introduction.

The aim of the course is to introduce the basic tools and techniques of modern complex algebraic geometry.
We will do this by giving an answer to the following question:

"When is a complex manifold an algebraic variety?"

A "complex manifold" is, roughly speaking, a topological space locally homeomorphic to $\mathbb{C}^n$, on which holomorphic functions can be defined; while an "algebraic variety" is a subset of a complex projective space golbally defined as the common zero locus of some homogeneous polynomials; the "is" in the question can be rewritten as: there exists a holomorphic map which "embeds" the complex manifold into a complex projetive space.

The answer was given by Kunihiko Kodaira with his two famous theorems, Kodaira Vanishing and Embedding: for this work he was awarded the Fields medal in 1954. The result, although derived by applying trascendental methods (i.e. related to holomorphic functions,) is toplogical in nature, and concerns the existence of a particular differential form.

Most of the material is taken from the initial chapters of Griffiths, P., Harris, J., "Principles of Algebraic Geometry".

Topics.

Modern algebraic geometry is best expressed in the languadge of sheaf cohomology, so we will introduce sheaves, pre-sheaves, and their cohomologies. We will then study the sheaf of holomorphic functions, first on $\mathbb{C}^n$ and then on complex manifolds, which we will define. We will introduce meromorphic functions on compex manifolds which will lead to the definition of divisors. Divisors on a complex manifold are closely related to line bundles (a particular type of vector bundles) on that manifold, which in turn are related to holomorphic maps from that manifold to a projective space. We will then define and study additional structures on a complex manifold (or on a vector bundle): differential forms, Hermitian metrics, Kälher metrics, connections, curvatures. This will bring us, after a brief introduction of Hodge theory and blow-ups, to the formulation and proof of Kodaira's theorems.

Practical Information.

The course will last 42 hours, and it will take place in the second semester. The course will be taught in Italian. The course grade will be based on a presentation by the student of a topic related to the course and choosen by the student. The prerequisites for this course are the basic notions of holomorphic functions in one variable, general topology, and algebraic topology.