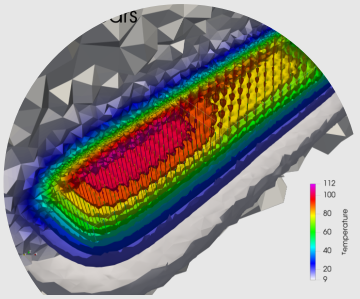

# Numerical Methods for PDEs

Metodi Numerici per le PDE

## Instructor Info


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Office Hrs: Thur. 8-10am

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## Course Info

Related courses:

- Calcolo Scientifico
- Metodi Numerici per le ODE
- Istituzioni di Analisi Numerica

2 Days a week

2h per lecture/42h

6 CFU

## Software and HPC

High-Performance Computing (HPC) plays a crucial role in the solution of Partial Differential Equations (PDEs). PDE solvers can be computationally intensive, especially for large-scale simulations. HPC clusters and supercomputers enable parallelization of these computations, where different parts of the problem are solved simultaneously. The course will touch on some of these issues and will discuss the writing of prototype codes and briefly the use of specific libraries suitable for these purposes; see, e.g., FEniCSx.

## Overview

Partial differential equations (PDEs) have a rich history and arise from various scientific and engineering disciplines. They are fundamental in describing how physical quantities such as temperature, pressure, concentration, and electromagnetic fields evolve over time and space. PDEs play a critical role in engineering disciplines. For example, they are used to model stress and strain in materials (in structural engineering), fluid flow (in aerospace and mechanical engineering), and electromagnetic fields (in electrical engineering). PDEs are applied in biology to model various processes. They are used in population dynamics to study the spread of species, in reaction-diffusion equations to model chemical reactions, and in neuroscience to simulate the propagation of electrical signals in neurons. PDEs are used in economics to describe how economic variables change over time and space. PDEs are often challenging to solve analytically, especially for complex systems and boundary conditions. Numerical methods provide a practical and efficient approach to approximate solutions for a wide range of PDEs. For this, it is necessary to use numerical methods to obtain approximations of their solutions that are

1. corresponding to the analytical properties of the underlying models,
2. easy to calculate,
3. accurate enough for its intended use in the reference application.

## Reading material

Brenner, Susanne C.; Scott, L. Ridgway. The mathematical theory of finite element methods. Third edition. Texts Appl. Math., 15. Springer, New York, 2008. xviii+397 pp. ISBN:978-0-387-75933-3.

## Exam methods

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|---|---|
| 60% | Seminar discussion of a scientific article agreed with the teacher or implementation of a method for the solution of a model PDE, |
| 40% | Oral exam on the course topics. |

## Topics covered by the course

Of all the possible numerical methods for the solution of PDEs, the course will focus on the use of the so-called Finite Element Methods. While PDEs are usually formulated in their strong form, i.e., involving derivatives of the unknown function $u$ with respect to the independent variables (typically time and space) and possibly source terms, this approach is not particularly flexible to approach its numerical solution. Therefore, a commonly used approach is to pass to the so-called weak formulation. In this way, one seeks a solution in a broader space of functions that may not necessarily be differentiable. Instead of requiring the PDE to hold exactly, it is required to hold in an integral sense, over the entire problem domain. Within this framework, FEM methods break down the complex physical domain over which the new integral formulation is defined into smaller, simpler subdomains called finite elements. These elements are typically triangles, quadrilaterals, tetrahedra, or hexahedra in 2D or 3D space. Within each finite element, the behavior of the physical quantity of interest (e.g., temperature, stress, displacement) is approximated using piecewise functions, often polynomials.

The course will start by recalling the useful results from the theory of Sobolev spaces that are needed to obtain the Weak Formulation of an Elliptic Boundary Value Problem. Having fixed the theoretical framework the course will move on to discussing the construction of a Finite Element Space. To make the analysis rigorous, we will discuss the theory of polynomial approximation in Sobolev Spaces and use it to obtain convergence theorems for different cases. To extend the range of applicability of the theory to PDEs coming from fluid dynamics problems, we will introduce variational crimes and the usage of mixed methods. To complement the theory and give practical tools, we will address both the implementation of some of the routines necessary for the construction of the discretization and the use of programming languages based on the Unified Form Language for the solution of these problems.