Gruppi e Rappresentazioni

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2024/25 - 2nd Semester

1 Introduction

The course "Gruppi e Rappresentazioni" is an introductory course on the theory of group representations. Representations, which are linear actions of groups, are a special kind of group action that arises naturally in various contexts, both within and outside mathematics, particularly in the presence of symmetry.

The theory of representations is remarkably rich and powerful. One of the first main results of this course will be Maschke's theorem, which states that representations of finite groups over the real or complex field decompose as a direct sum of irreducible representations. As a result, we can focus on studying and classifying irreducible representations, which can be effectively done by examining their character, i.e., their trace. We will also associate a special ring with any group, called the group algebra, and explore how many properties of representations can be studied from the perspective of non-commutative algebra.

In the second part of the course, we will explore the representation theory of symmetric groups in greater detail, which is particularly important and well-developed. Here, combinatorial objects such as Young tableaux play a crucial role in constructing and studying the irreducible representations.

In the final part, we will also connect this to the theory of the general linear group, which is deeply intertwined with the symmetric group through Schur–Weyl duality.

2 Practical Information

The course will last 48 hours, and it will take place in the second semester. Throughout the course, the students will have the possibility to submit the solutions to some exercises that will be assigned in class: if they do so, that will be counted as part of the exam, the rest of which will consist in a seminar to be presented within the summer exam session; if they do not, they will have to take an oral exam instead, which will include questions on both theory and exercises. The prerequisites for this course are basic linear algebra and basic theory of finite groups. For any further question, please contact michele.dadderio@unipi.it or leonardo.patimo@unipi.it.