Teoria dei Campi e Teoria di Galois

078AA MAT-L/WMA

A.A. 2024/25 - Second semester

Teachers: Andrea Bandini - Ilaria Del Corso

The aim of the course is to provide a broad introduction to the field theory, covering both finite and infinite extensions in any characteristic. The course will focus specifically on Galois theory, Galois modules, and Kummer theory. These topics appear in various areas of Mathematics and are crucial tools for students who intend to pursue further studies in Algebraic Number Theory, Arithmetic Geometry, Algebraic Geometry, *etcetera*.

Main Features

The course begins with a recap of the theory of finite extensions, which is then generalized to the infinite case, emphasizing the role of separability. We then examine the decomposition of a generic extension into a separable part and a purely inseparable part, focusing on the properties of these two types of extensions.
For the study of Galois extensions, we introduce profinite groups as models for Galois groups and use their topological properties to prove the Fundamental Theorem of (infinite) Galois Theory. In the final part of the course, we present Galois modules and employ a basic cohomological approach (via Hilbert's Theorem 90) to describe abelian Galois extensions of certain fields using Kummer theory.
All concepts will be illustrated with examples (and counter-examples) of fields in various characteristics. Applications will include the solvability of equations by radicals and some cases of the inverse Galois problem.

Practical Information

- This 48 hours course (6 CFU) will be held in the second semester, and is intended for students of both the Bachelor and Master programs in Mathematics.
- A few prerequisites are needed, mainly the contents of the Arithmetic and Algebra 1 courses, and also a basic knowledge of Topology and Linear Algebra.
- The final exam consists of an oral test.
- For further information please visit the website of the course on the ECTS catalogue of the University of Pisa.