Istituzioni di Algebra

Instructor: Davide Lombardo

2025-2026
(first semester)

1 Contents

The course will discuss some more advanced topics in commutative algebra that are not covered in the Algebra 1 and 2 courses, the foundations of homological algebra, and some applications of the latter to commutative and non-commutative algebra (including for example the homological characterisation of regularity and applications in group theory). Many of these topics are prerequisites for a serious study of algebraic geometry, number theory, and algebraic topology, and – without going into too much detail – we will try to emphasise the connections as much as possible.

1. Commutative algebra: integral extensions, going-up and going-down theorems, Krull's Hauptidealsatz, dimension theory, regular sequences and regular rings, completions. If time permits: algebraic notions of smoothness and differentials.

2. Homological algebra: complexes, derived functors, Ext and Tor, group cohomology. Homological dimension, Hilbert's syzygy theorem. If time permits: introduction to spectral sequences.

2 Practical information

The course lasts 72 hours and takes place in the first semester. The final examination can be taken in two ways:

1. written and oral exam on the same day;

2. alternatively, students may submit homework throughout the semester (there will be four assignments); the final exam will then consist of a short written exam (asking for the solution of an exercise that the student has already submitted) and an oral exam.

Prerequisites for Istituzioni di Algebra are the compulsory courses of the bachelor's degree, as well as Algebra 2. Some familiarity with algebraic geometry (at the level of Elementi di Geometria Algebrica) may be useful but is not necessary.

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