Low-Dimensional Topology and Geometry (288AA)

Paolo Lisca

General Description of the Course

This is an advanced course aimed at final-year master's students and PhD students. The goal of the course is to develop some relevant tools for the study of the topology of smooth low-dimensional manifolds, with particular emphasis on four-dimensional ones.

Course Contents

Basic Results on 3-Manifolds

Heegaard splittings. Reidemeister-Singer theorem. Heegaard diagrams. Mapping class group. Dehn-Lickorish theorem. Presentations of 3-manifolds via surgery.

Overview of 4-Manifolds

Intersection form and its properties. Examples. Classification of unimodular forms. Freedman's results on topological 4-manifolds and those of Rohlin, Donaldson, and Furuta on smooth 4-manifolds. Existence of exotic 4-manifolds.

Handle Decompositions

Existence of handle decompositions. Reordering, cancellation, creation, and sliding of handles. Cerf's theorem. Determination of topological invariants from a handle decomposition. Kirby diagrams and handle moves. Surgery and 1-handles.

Construction of Exotic Structures

Thom's conjecture. Adjunction inequality. Contact structures on 3-manifolds. Stein 4-manifolds and their Kirby diagrams. Adjunction inequality for Stein 4-manifolds. Akbulut exotic manifolds.

Optional Topics

At the instructor's discretion and time permitting, one or more of the following topics may also be covered. Akbulut caps. Akbulut-Kirby conjecture and Yasui's counterexamples. Abe's conjecture and Miller-Piccirillo's counterexamples. Piccirillo's theorem on the Conway knot. Hayden's exotic disks.

Teaching Method

Traditional chalkboard lectures.

Prerequisites

Contents of the courses "Elementi di Topologia Algebrica" and "Istituzioni di Geometria".

Examination Method

Seminar presentation on a research paper.

Recommended Texts

- S. Akbulut, "4-Manifolds", Oxford Graduate Texts in Mathematics, Oxford UP.
- R.E. Gompf, A.I. Stipsicz, "4-Manifolds and Kirby Calculus", Graduate Studies in Mathematics, American Mathematical Society.
- V.V. Prasolov, A.B. Sossinsky, "Knots, Links, Braids and 3-Manifolds", American Mathematical Society.